18.8.12

Centre Of Mass And Rotation Motion

* **Centre of Mass:**

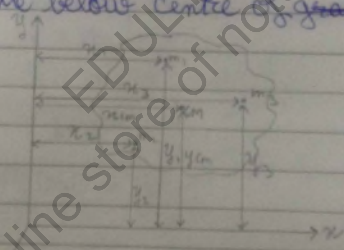
A hypothetical point at which whole mass of body is supposed to be concentrated is know as centre of Mass of body

→ It can lie outside ~~E~~ of body where mass is ~~not~~ distributed.

→ For regular shapes centre of mass coincide with geometrical centre.

→ The point about which a hanging body can be in Eq^m is know as C.O.G.

→ for small bodies C.O.G. & centre of Mass are same but for larger bodies centre of ~~mass~~ gravity is little below centre of ~~gravity~~ mass.



$$\vec{x}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2 + \dots + m_n\vec{x}_n}{m_1 + m_2 + \dots + m_n}$$

$$\vec{y}_{cm} = \frac{m_1\vec{y}_1 + m_2\vec{y}_2 + \dots + m_n\vec{y}_n}{m_1 + m_2 + \dots + m_n}$$

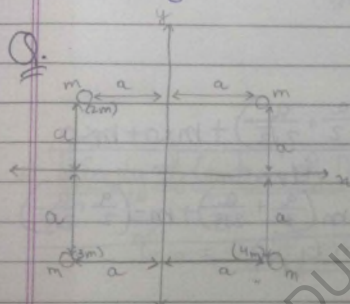
$$\vec{Z}_{cm} = \frac{m_1\vec{Z}_1 + m_2\vec{Z}_2 + \dots + m_n\vec{Z}_n}{m_1 + m_2 + \dots + m_n}$$

→ Centre of Mass of a body/system is depend upon choice of coordinate system

→ x, y & z are vector quantities so, sign convention is considered.

→ Mass is a scalar quantity so, it doesn't depend upon the position of particle, but if a particle is added then its mass is taken as +ve, when mass is removed from a body then taken as -ve.

Q.



find centre of mass for system.

→ (0,0)

If, m, 2m, 3m & 4m.

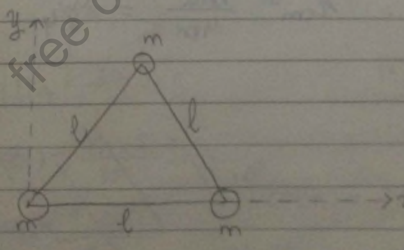
$$x_{cm} = \frac{ma + 2m(-a) + 3m(-a) + 4m(a)}{10m}$$

$$x_{cm} = 0$$

$$y_{cm} = \frac{ma + 2ma + 3m(-a) + 4m(-a)}{10m}$$

$$y_{cm} = \frac{-4}{10}$$

Q.

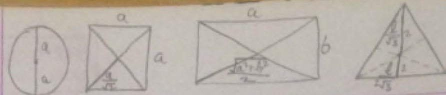


$$x_{cm} = \frac{m\times 0 + ml + m\frac{l}{2}}{3m}$$

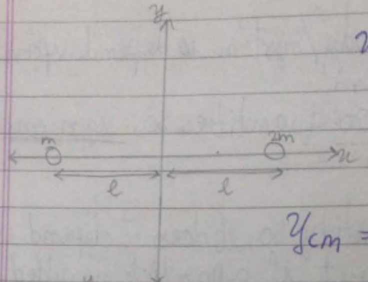
$$x_{cm} = \frac{l}{2}$$

$$y_{cm} = \frac{m\times 0 + m\times 0 + m\sqrt{l^2 - \frac{l^2}{4}}}{3m}$$

$$= \frac{\sqrt{4l^2 - l^2}}{2\times 3} = \frac{\sqrt{3l^2}}{6} = \frac{l\sqrt{3}}{6} = \frac{l}{2\sqrt{3}}$$



Q.

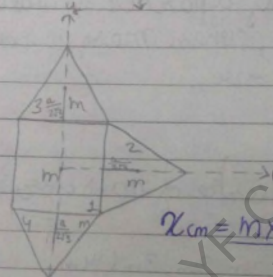


$$x_{cm} = \frac{-m\times l + 2ml}{2m}$$

$$= \frac{ml}{2m} = \frac{l}{2}$$

$$y_{cm} = 0$$

Q.

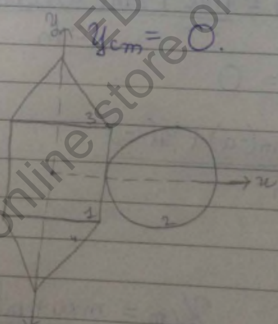


$$x_{cm} = \frac{m\times 0 + m\left(\frac{a}{2} + \frac{a}{2\sqrt{3}}\right) + m\times 0 + m\times 0}{4m}$$

$$y_{cm} = \frac{m\times 0 + m\times 0 + m\left(\frac{a}{2} + \frac{a}{2\sqrt{3}}\right) + m - \left(\frac{a}{2} + \frac{a}{2\sqrt{3}}\right)}{4m}$$

$$y_{cm} = 0.$$

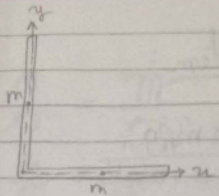
Q.

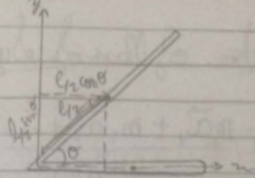


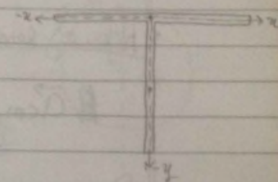
$$y_{cm} = 0$$

$$x_{cm} = \frac{m\times 0 + ma + m\times 0 + m\times 0}{4m}$$

$$x_{cm} = \frac{ma}{4m} = a/4$$







$$x_{cm} = \frac{m\frac{l}{2} + m\times 0}{2m} = \frac{l}{4}$$

$$y_{cm} = \frac{l}{4}$$

$$x_{cm} = \frac{m\frac{l}{2} + m\frac{l}{2}\cos\theta}{2m} = \frac{l}{4}(1+\cos\theta)$$

$$y_{cm} = \frac{m\times 0 + m\frac{l}{2}\sin\theta}{2m} = \frac{l}{4}\sin\theta$$

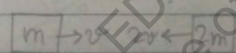
$$x_{cm} = \frac{m\times 0 + m\times 0}{2m} = 0$$

$$y_{cm} = -\frac{l}{4}$$

* Velocity of Centre of Mass of System -

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \text{-----} + m_n\vec{v}_n}{m_1 + m_2 + \text{----} + m_n}$$

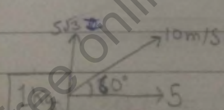
Q.

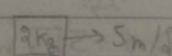


find vel. of centre of mass of system.

$$\Rightarrow v_{cm} = \frac{mv - 2m2v}{3m} = \frac{mv - 4mv}{3m} = \frac{-3mv}{3m} = -v$$

Q.





$$\cancel{v_{cm(x)} = \frac{m\times 5}{m} = 5\,m/s}$$

$$\cancel{v_{cm(y)} = \frac{1\times 5\sqrt{3}}{1}}$$

$$\cancel{v_{cm} = \frac{2\times 5}{2}} = 5\,m/s$$

$$v_{cm(x)} = \frac{1\times 5 + 2(5)}{3} = \frac{5+10}{3} = 5\,m/s$$

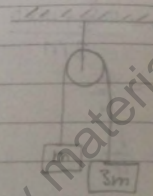
$$v_{cm(y)} = \frac{1\times 5\sqrt{3} + 2\times 0}{3} = \frac{5}{\sqrt{3}}\,m/s$$

$$v_{cm} = \sqrt{25 + \frac{25}{3}} = \frac{10}{\sqrt{3}}\,m/s$$

- **Acc. of centre of Mass of System:**

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n}{m_1 + m_2 + \cdots + m_n}$$

Q. Find acc. of centre of mass of system.



→

$$a = \frac{3mg - mg}{4m}$$

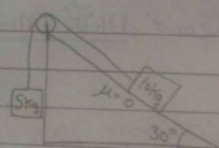
$$= \frac{g}{2} = 5\,m/s^2$$

$$a_{cm} = \frac{m\times 5 + 3m(-5)}{4m}$$

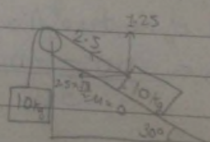
$$= \frac{-10\cancel{m}}{4\cancel{m}} = -2.5\,m/s^2$$

Q. $a = 0$

$\therefore a_{cm} = 0$



Q.



$$a = \frac{100-50}{20} = 2.5\,m/s^2$$

$$a_{cm(x)} = \frac{10\times 0 + 10\times -1.25\sqrt{3}}{2\cancel{0}}$$

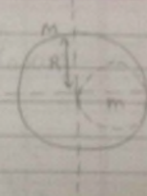
$$= -0.675\sqrt{3}\,m/s^2$$

$$a_{cm(y)} = \frac{10\times(-2.5) + 10\times 1.25}{20}$$

$$= \frac{-25+12.5}{20} = -\frac{12.5}{20} = -0.625\,m/s^2$$

$$a_{cm} = 0.625\sqrt{3+1} \Rightarrow = 1.25\,m/s^2$$

Q.



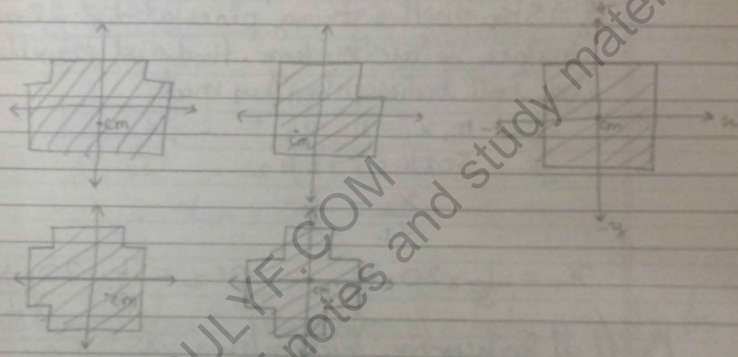
$$\frac{M}{\pi R^2} = \frac{m}{\pi \left(\frac{R}{2}\right)^2}$$

$$\frac{M}{R^2} = \frac{4m}{R^2}$$

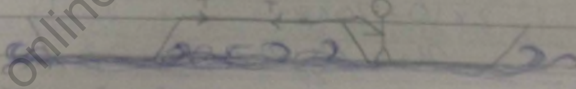
$$m = \frac{M}{4}$$

$$x_{cm} = \frac{M \times 0 + \left(-\frac{M}{4}\right)\frac{R}{2}}{M + \left(-\frac{M}{4}\right)} = -\frac{R}{6}$$

Q.



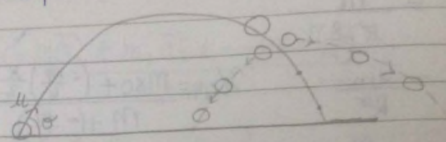
Note - If there is no external force acting on system then centre of mass doesn't changes its position or vel. of centre of mass remains zero.



$$\Sigma m x = \text{Const.}$$

$$m_1 x_1 = m_2 x_2$$

- If a projectile is thrown with some speed at any angle and it is exploded into many particles in b/w of its path, then particle moves in diff. directions with diff. speeds, but centre of mass follows same parabolic path

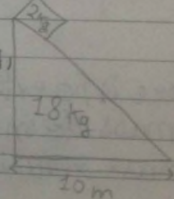


Q. A man of mass 50 kg is pulling another body of 25 kg with a string place at dist. 20 m. If surface is frictionless, find dist. travelled by both bodies when they meet.

→
$$m_1 x_1 = m_2 x_2$$
$$50 \times x_1 = 25 \times x_2$$
$$\frac{x_1}{x_2} = \frac{1}{2}$$

$x_1 = \frac{1}{3} \times 20 = \frac{20}{3} m$ $\quad x_2 = \frac{2}{3} \times 20 = \frac{40}{3} m$

Q. If all surfaces are frictionless, find dist. travelled by body of 18 kg, when block slides down to bottom.



→
$$m_1 x_1 = m_2 x_2$$
$$2 \times 10 = 20 \times x_2$$
$$x_2 = \frac{2 \times 10}{20} = 1 m$$

22/8/12

Centre of mass for non-uniformly distributed mass system (density is non-uniform)

$$x_{cm} = \frac{\int x\,dm}{\int 1\,dm}$$

Q. A rod of length l & mass m has non-uniform density $\rho = \rho_0 ($ Find CM ~~of m~~ of rod from first end.

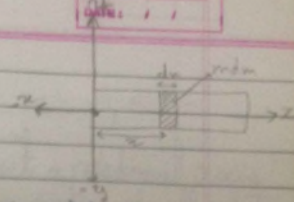
$$x_{cm} = \frac{\int x\,dm}{\int 1\,dm}$$

$$\because \frac{dm}{dx} = \frac{M}{l}$$

$$dm = \frac{M}{l}dx$$

$$dm = \frac{Al}{l}\rho_0\left(1+\frac{x}{l}\right)dx$$

$$dm = A\rho_0\left(1+\frac{x}{l}\right)dx$$



$$\therefore x_{cm} = \frac{\int x\,dm}{\int 1\,dm} = \frac{\int_0^l x A\rho_0\left(1+\frac{x}{l}\right)dx}{\int_0^l 1 A\rho_0\left(1+\frac{x}{l}\right)dx}$$

$$= \frac{\int_0^l\left(x+\frac{x^2}{l}\right)dx}{\int_0^l\left(l+\frac{x}{l}\right)dx} = \frac{\left[\frac{x^2}{2}+\frac{x^3}{3l}\right]_0^l}{\left[x+\frac{x^2}{2l}\right]_0^l}$$

$$= \frac{\left[\frac{l^2}{2}+\frac{l^3}{3l}\right]}{\left(l+\frac{l}{2l}\right)^2} + \frac{\left\{\frac{l^2}{2}+\frac{l^2}{3}\right]}{\left(l+\frac{l}{2}\right)} = \frac{\frac{5l^2}{6}}{\frac{3l}{2}}$$

$$\therefore x_{cm} = \frac{5l}{9}$$

* **Rotational Motion:-** The motion of a body is said to be rotational if:
  1. Each moving particle moves in circular motion about any line taken as centre
  2. Plane of circulation of all particles is $\perp$ to that line
  3. Particles at that line should be at rest.
  4. The line about which all particles circulate is known as Axis of Rotation.

* **Newton's first law Rotatory motion:-**

A body remains in same state of rest or rotatory motion until an external torque is applied on it.