

3. Motion in a straight line

IMPORTANT POINTS

- Study of motion of objects along a straight line is known as rectilinear motion.
- If a body does not change its position with time it is said to be at rest. If it changes its position with time it is said to be in motion. The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight line, position to the right of the origin is taken as positive and to the left as negative.
- Path length is defined as the total length of the path traversed by an object.
- Displacement is the change in position : $\Delta x = x_2 - x_1$, Path length is greater than or equal to the magnitude of the displacement between the two positions
- An object is said to be in uniform motion in a straight line if its displacement is equal in equal intervals of time. Otherwise the motion is said to be non-uniform.
- Average velocity is the ratio of the displacement and time interval in which the displacement occurs.

$$\bar{V} = \Delta x / \Delta t$$

On an x-t graph, the average velocity over a time interval is the slope of the line connecting the initial and final positions corresponding to that interval.

- Average Speed is ratio of the total path length traversed and the corresponding time interval. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time.
- Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$V_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \bar{V} = \lim_{\Delta t \rightarrow 0} \Delta x / \Delta t = dx/dt$$

The velocity at a particular instant is equal to the slope of the tangent drawn on position –time graph at that instant.

- Average acceleration is the change velocity divided by the time interval during which he change occurs.

$$a = \Delta V / \Delta t$$

- Instantaneous acceleration is defined as the limit of the average acceleration as the time interval Δt goes to zero.

$$\mathbf{a} = \lim_{\Delta t \rightarrow 0} (\mathbf{a}) = \lim_{\Delta t \rightarrow 0} (\Delta \mathbf{V} / \Delta t) = d\mathbf{v} / dt$$

$$\Delta t \rightarrow 0 \quad \Delta t \rightarrow 0$$

The acceleration of an object at a particular time is the slope of the velocity- time curve at that instant of time. For uniform motion, acceleration is zero and x-t graph is a straight line inclined to the time axis. And v-t graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola, while the v-t graph is a straight line inclined to the time axis.

- The area under the velocity- time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.
- For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x , time taken t , initial velocity u , final velocity v and acceleration are related by a set of simple equations called kinematic equations of motion.

(i) $\mathbf{V} = \mathbf{u} + \mathbf{a}t$

(ii) $\mathbf{X} = \mathbf{u}t + \frac{1}{2} \mathbf{a} t^2$

(iii) $\mathbf{V}^2 - \mathbf{u}^2 = 2 \mathbf{a} x$

Solve the following. Each question carries 1 mark.

1. Distinguish between scalar and vector quantities. Give an example in support of each.
2. Define speed. How is it different from velocity?
3. Define uniform velocity and variable velocity?
4. Plot the velocity- time graph for a uniform motion. What does the area under the graphs indicate?
5. Write any two equations of motion of a body moving with uniform acceleration.
6. Plot a velocity- time graph for a body moving with uniform acceleration.
7. Plot position – time graph for a body having uniformly retarded motion
8. What does the speedometer of car indicate?

9. Two cars are running at velocities of 60 km/hr and 45 km/hr respectively. Calculate the relative velocity of car A, if (i) they are both travelling eastwards; and (ii) car A is travelling eastwards and car B is travelling westwards.

10. A body goes from A to B with a velocity of 40 m/sec, and comes back from B to A with a velocity of 60 m/sec. What is the average velocity of the body during the whole journey.

Answer the following questions each question carries 2 marks

11. A player throws a ball upwards with an initial speed of 39.2 m/sec.

(a) What is the direction of acceleration during the upward motion?

(b) Find the velocity and acceleration of the ball at the highest point.

(c) Find the height through which the ball rises, and the time after which it returns to the player's hands.

12. From the top of a tower 100m height, balls is dropped, and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/sec. find when and where the two balls meet. Take $g = 9.8 \text{ m/sec}^2$

13. The distance travelled by a body is found to be directly proportional to the square of time. Is the body moving with uniform velocity or with uniform acceleration?

14. The displacement (x) of a particle moving in one dimension, under the action of a constant force related to time t by the relation $t = \sqrt{x} + 3$ where x is in meters, and t is in seconds. Find the displacement of the particle when its velocity is zero.

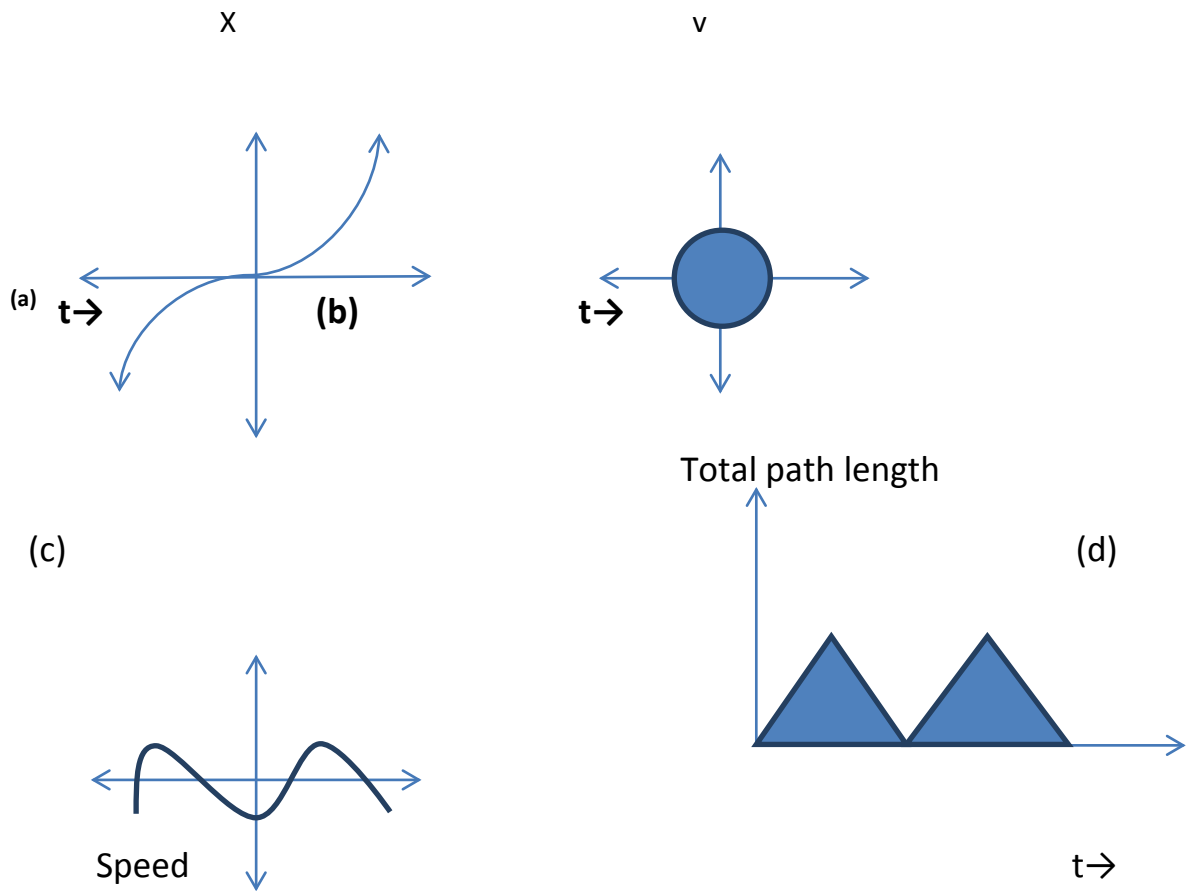
15. Can a body have zero velocity, and finite acceleration?

16. Can a body have constant speed, but a varying velocity?

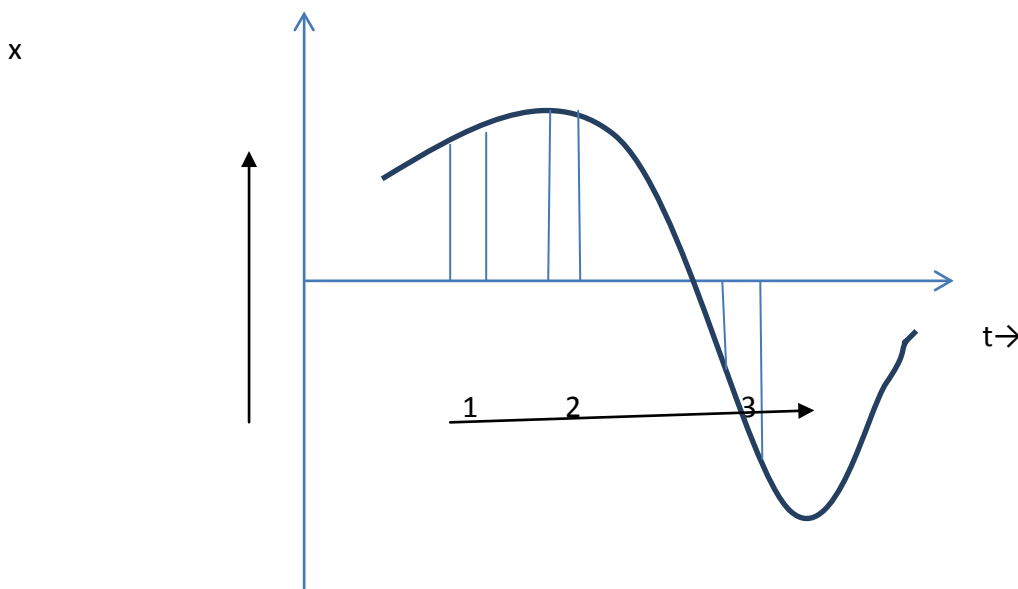
17. Can a body have constant velocity, but a varying speed?

18. Why is it that a parachute descends slowly whereas a stone dropped from the same height falls rapidly?

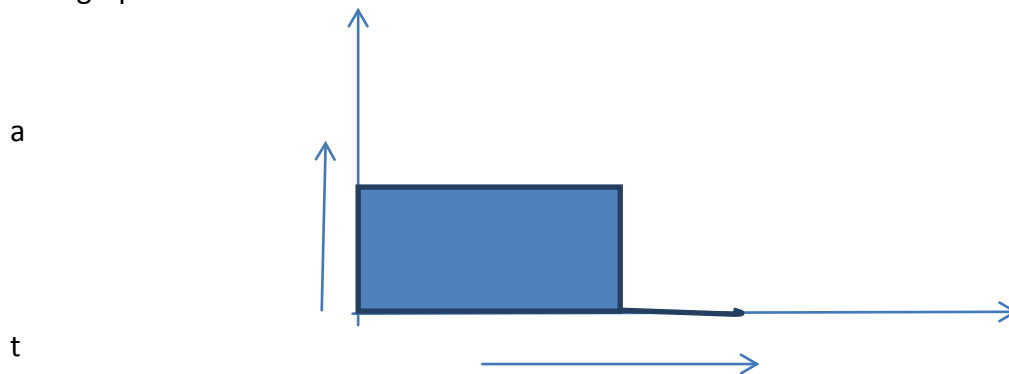
19. Look at the graphs (a) to (d) in fig carefully and state with reasons which of these cannot possibly represent one dimensional motion of a particle.



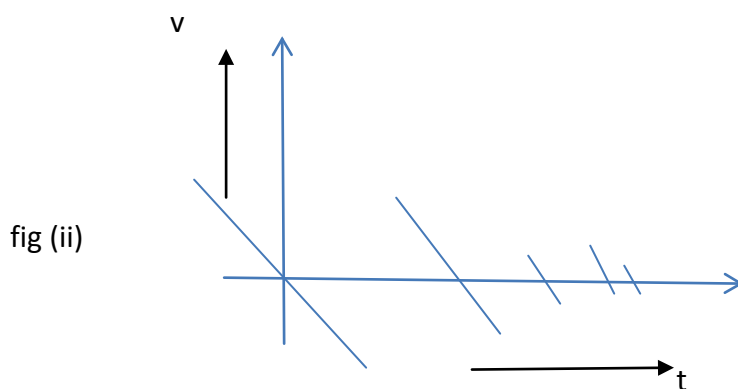
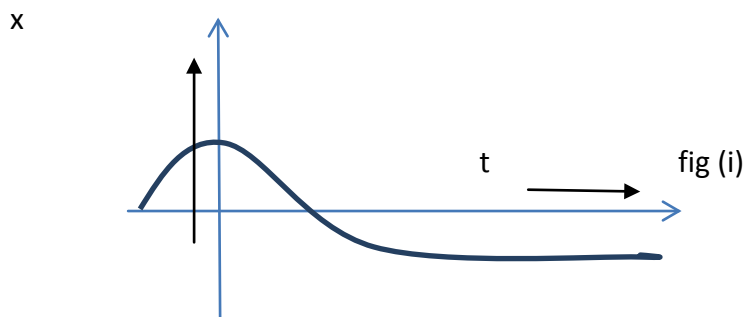
20. Fig gives the x - t plot of a particle in one dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval



21. The acceleration –time graph for a body is shown below. Plot the corresponding velocity-time graph



21. Suggest a suitable physical situation for each of the following graphs



22. Give the equations of motion of a body falling under gravity. Also give the graphs showing the variation of (i) acceleration of a body with time

(ii) Velocity of a body with time

(iii) Distance with time in case of a freely falling body.

23. Discuss the motion of an object under free-fall.

24. Derive a relation between the position and time for a particle moving with uniform acceleration.

25. Derive a relation for the distance covered in the n th second by a uniformly accelerated body.

26. Show that when a body has uniformly accelerated motion, the distance covered in a certain interval is equal to the area under the velocity-time graph for that time interval.

27. A body is moving with uniform acceleration its velocity after 5 seconds is 25 m/sec and after 8 seconds is 34 m/sec. Calculate the distance it will cover in the 10th second.

28. The speed of a train increases at a constant rate α from zero, to v , and then remains constant for an interval, and finally decreases to zero at a constant rate β . If L be the total distance described, prove that the total time taken is

$$(L/v) + (v/2) (1/\alpha + 1/\beta)$$

29. A body moving with a uniform acceleration describes 12 m in the third second of its motion and 20m in the fifth second. Find the velocity after 10 seconds.

Description of Motion in Two and Three Dimensions

Main Points:

1. Scalar quantities with magnitudes only. Examples are distance, speed mass and temperature.

2. Vector quantities with magnitude and direction both. Examples are displacement, velocity and acceleration. They obey special rules of vector algebra.

3. A vector ' \mathbf{A} ' multiplied by a real number ' λ ' is also a vector, whose magnitude is ' λ ' times the magnitude of the vector ' \mathbf{A} ' and whose direction is same or opposite depending upon whether ' λ ' is positive or negative.

4. Two vectors \mathbf{A} and \mathbf{B} may be added graphically using head to tail method or parallelogram method.

5. Vector addition is **commutative**:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

It also obeys the **associative law**:

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$$

6. A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction. It has the properties:

$$\mathbf{A} + \mathbf{0} = \mathbf{A}$$

$$\lambda \mathbf{0} = \mathbf{0}$$

$$\overline{\mathbf{0}} \cdot \overline{\mathbf{A}} = \overline{\mathbf{0}}$$

7. The subtraction of vector **B** from **A** is defined as the sum of **A** and **-B**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

8. A vector **A** can be resolved into components along two given vectors **a** and **b** lying in the same plane:

$\mathbf{A} = \lambda \mathbf{a} + \mu \mathbf{b}$ where λ and μ are real numbers.

9. A unit vector associated with a vector **A** has magnitude one and is along the vector **A** :

$\hat{n} = \frac{A}{|A|}$ The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are vectors of unit magnitude and point in the direction of x, y, and z- axes respectively in a right handed coordinate system.

10. A vector 'A' can be expressed as

$\mathbf{A} = A_x \hat{i} + A_y \hat{j}$ where A_x, A_y are components along x- and y- axes . If vector **A** makes an angle Θ with the x- axis, then $A_x = A \cos\Theta$, $A_y = A \sin\Theta$ and $A = \sqrt{A_x^2 + A_y^2}$,

$$\tan \Theta = A_y/A_x.$$

11. Vectors can be conveniently added using analytical method. If sum of two vectors 'A' and 'B', that lie in x-y plane is 'R', then:

$$\mathbf{R} = R_x \hat{i} + R_y \hat{j}, \text{ where } R_x = A_x + B_x \text{ and } R_y = A_y + B_y$$

12. The position vector of an object in x-y plane is given by $\mathbf{r} = x \hat{i} + y \hat{j}$ and the displacement from position **r** to position **r'** is given by

$$\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$$

$$= (x' - x)\hat{i} + (y' - y)\hat{j}$$

$$= \Delta x \hat{i} + \Delta y \hat{j}$$

13. If an object undergoes a displacement Δr in time Δt , its average velocity is given by

$V = \Delta r / \Delta t$. The velocity of an object at time t is the limiting value of the average velocity as Δt tends to zero..

$V = \lim_{\Delta t \rightarrow 0} \Delta r / \Delta t = dr / dt$.It can be written in unit vector notation as

$\Delta t \rightarrow 0$

$$V = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \quad \text{where } V_x = dx/dt, V_y = dy/dt, V_z = dz/dt$$

When position of an object is plotted on a coordinate system v is always tangent to the curve representing the path of the object.

14. If the velocity of an object changes from v to v' in time Δt , then its average acceleration is given by $\bar{a} = (v - v') / \Delta t = \Delta v / \Delta t$

The acceleration a at any time t is the limiting value of \bar{a} as $\Delta t \rightarrow 0$

$$a = \lim_{\Delta t \rightarrow 0} \Delta v / \Delta t = dv / dt$$

$\Delta t \rightarrow 0$

In component form, we have $a = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

Where $a_x = dv_x / dt, a_y = dv_y / dt, a_z = dv_z / dt$

15. If an object is moving in a plane with constant acceleration $a = \sqrt{a_x^2 + a_y^2}$

And its position vector at time $t = 0$ is r_0 , then at any other time t , it will be at a point given by

$$r = r_0 + V_0 t + \frac{1}{2} a t^2$$

and its velocity is given by :

$$V = V_0 t + at \text{ where } V_0 \text{ is the velocity at time } t = 0$$

In component form

$$X = x_0 + V_{0x} t + \frac{1}{2} a_x t^2$$

$$Y = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$$

$$V_x = V_{0x} + a_x t$$

$$V_y = V_{0y} + a_y t$$

Motion in a plane can be treated as superposition of two separate simultaneous one dimensional motions along two perpendicular directions.

16. An object that is in flight after being projected is called a projectile. If an object is projected with an initial velocity V_0 making an angle θ_0 with x-axis and if assume its initial position to coincide with the origin of the coordinate system, then the position and velocity of the projectile at time t are given by

$$X = (V_0 \cos \theta_0) t$$

$$Y = (V_0 \sin \theta_0) t - \frac{1}{2} g t^2$$

$$V_x = V_{0x} = V_0 \cos \theta_0$$

$$V_y = V_0 \sin \theta_0 - g t$$

The path of a projectile is parabolic and is given by

$$Y = (\tan \theta_0) x - \frac{g x^2}{2 V_0^2 \cos^2 \theta_0}$$

The maximum height that a projectile attains is

$$H_m = \frac{(V_0 \sin \theta_0)^2}{2g}$$

The time taken to reach this height is

$$t_m = \frac{V_0 \sin \theta_0}{g}$$

The horizontal distance travelled by a projectile from its initial position to the position it passes $y = 0$ during its fall is called the RANGE, R of the projectile. It is:

$$R = \frac{V_0^2 \sin 2\theta_0}{g}$$

Projectile motion

Motion along vertical direction

Motion along horizontal direction

force of gravity acting on the body

Equations of motion of a body moving with constant acceleration

No force in the horizontal direction and hence horizontal velocity remains constant.

17. When an object follows a circular path at constant speed. The motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2/R$. The direction of a_c is always towards the centre of the circle.

The angular speed is the rate of change of angular distance. It is related velocity v by

$v = \omega R$. The acceleration is $a_c = \omega^2 R$.

- If T is the time period of revolution of the object in circular motion and v is the frequency then we have $\omega = 2\pi v = 2\pi/R$ $a_c = 4\pi^2 v^2 R$
- Centripetal force is the name given to the force that provides inward radial acceleration to a body in circular motion. We should always look for some material force like tension, gravitational force, electrical force, friction etc. as the centripetal force.

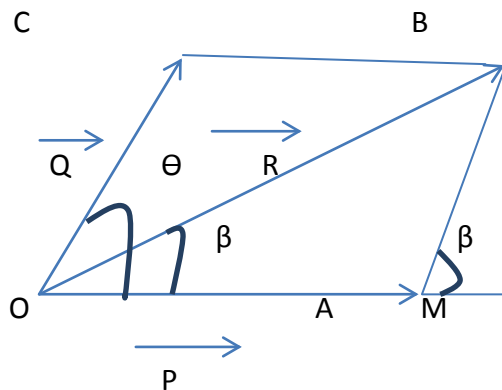
18. The path length traversed by an object between two points is not the same as the magnitude of displacement always. The displacement depends only on the end points; whereas the path length depends on the actual path. The two quantities are equal only if the object does not change its direction during the course of its motion. In all other cases, the path length is greater than the magnitude of displacement.

19. The average speed of an object is greater than or equal to the magnitude of the average velocity over a given interval of time. The two are equal only if the path length is equal to the magnitude of the displacement.

Answer the following

1. When are two vectors equal?
2. What are co-initial and collinear vectors?
3. When are two vectors equal?
4. State the triangle law of vector addition
5. State the parallelogram law of vector addition.
6. Using the parallelogram law of vectors, find the magnitude and direction of the resultant R in the following fig

Discuss cases for (i) $\theta = 0^\circ$ (ii) $\theta = 90^\circ$ (iii) $\theta = 180^\circ$



7. State three important properties of vector addition.
8. What do you mean by the null vector or zero vector.
9. Write properties of zero vector.
10. Define scalar product of two vectors. Give one example.
11. Define vector product of two vectors? Give one example.
12. Express the area of a parallelogram in terms of cross product of two vectors.
13. Define the following terms associated with projectile motion.
 - (i) Projectile (ii) trajectory (iii) time of flight (IV) maximum height (v) Horizontal Range
14. Give two examples of projectile motion.

15. Write two applications of projectile motion
16. At what angle with the vertical should a projectile be fired so that its range is maximum?
17. A projectile is fired with 'u' at an angle of projection θ . Write
- The equation of trajectory
 - Expression for the maximum height reached
 - Expression for the time of flight
 - Expression for the time taken to reach the maximum height.
 - expression for the horizontal range.
 - the other angle of projection for the same horizontal range.
18. Define average angular velocity. Write its units and dimensions.
19. Write the relation between angular velocity (ω), time period (T) and frequency (ν).
20. Write relation between angular acceleration and linear acceleration.
21. Find the min velocity for which the horizontal range is 39.2m
22. Prove that the Max horizontal range is 4 times the max height attained by a projectile.
23. Calculate the magnitude of the vector $\vec{r} = (3\hat{i} + 4\hat{j} + 7\hat{k})$
24. Prove that the vectors $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $(2\hat{i} - \hat{j})$ are perpendicular to each other.
25. A string can withstand a tension of 25 N. What is the greatest speed at which a body of mass 1 kg can be whirled in a horizontal circle using a 1m length of the string?
26. A bullet is fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to be fixed, and neglect air resistance. Take $g = 10 \text{ m/sec}^2$.
27. Show that the trajectory of a projectile is a parabola.
28. From the top of a building 19.6 m high, a ball is projected horizontally, after how long does it strike the ground? If the line joining the point of projection to the point, where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball?

29. A projectile is fired at an angle θ with the horizontal. Obtain the expressions for (i) the maximum height attained (ii) the time of flight (iii) the horizontal range.
30. What do you mean by centripetal force ? Derive an expression for the centripetal force.
31. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 seconds. (a) What is the angular speed, and the linear speed of the motion? (b) Is the acceleration vector a constant vector? What is its magnitude?
32. A cricket ball is thrown at a speed of 28 m/sec in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level and (c) the distance from the thrower to the point where the ball returns to the same level.
33. A particle starts from origin at $t=0$ with a velocity of $5.0 \hat{i}$ m/sec and moves in x-y plane under the action of a force which produces a constant acceleration of $(3.0\hat{i} + 2.0\hat{j})$ m/sec². What is the y-coordinate of the particle at the instant its x – coordinate is 84 m? (b) What is the speed of the particle at this time?