QUADRATIC EQUATIONS

★ INTRODUCTION

When a polynomial f(x) is equated to zero, we get an equation which is known as a polynomial equation. If f(x) is a linear polynomial than f(x) = 0 is called a linear equation. For example, $3x - 2 = 0$, $4t + \frac{3}{5} = 0$ etc. are linear equations. If f(x) is quadratic polynomial i.e., $f(x) = ax^2 + bc + c$, $a \neq 0$, then f(x) = 0 i.e., $ax_2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation. Such equations arise in many real life situations. In this chapter, we will learn about quadratic and various ways of finding their zeros or roots. In the end of the chapter, we will also discuss some applications of quadratic equations in daily life situations.

★ HISTORICAL FACTS

On clay tables dated between 1800 BC and 1600 BC, the ancient Babylonians left the earliest evidence of the discovery of quadratic equations, and also gave early methods for solving them.

Indian mathematician Baudhayana who wrote a Sulba Sutra in ancient India circa 8th century BC first used quadratic equations of the form : $ax^2 = c$ and $ax^2 + bx = c$ and also gave methods for solving them.

Babylonian mathematicians from circa 400 BC and Chinese mathematicians from circa 200 BC used the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula.

Euclid, a Greek mathematician, produced a more abstract geometrical method around 300 BC.

The first mathematician to have found negative solutions with the general algebraic formula was Brahmagupta (India, 7th century). He gave the first explicit (although still not completely general) solutions of the quadratic equations $ax^2 + bx = c$ as follows :

"To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value."

This is equivalent to : $$x = \frac{\sqrt{4ac + b^2} - b}{2a}$$

Muhammad ibn Musa al-Kwarizmi (Persia, 9th century) developed a set of formulae that worked for positive solutions.

Bhaskara II (1114-1185), an Indian mathematician-astronomer, solved quadratic equations with more than one unknown and is considered the originator of the equation.

Shridhara (India, 9th century) was one of the first mathematicians to give a general rule for solving a quadratic equation.

★ QUADRATIC EQUATIONS

A polynomial equations of degree two is called a quadratic equation.

Ex. $2x^2 - 3x + 1 = 0$, $4x - 3x^2 = 0$ and $1 - x^2 = 0$

General form of quadratic equations : $ax^2 + bx + c = 0$, where a,b,c, are real numbers and $a \neq 0$.

Moreover, it is general form of a quadratic equation in standard form.

Types of Quadratic Equations : A quadratic equation can be of the following types :

(i) $b = 0$, $c \neq 0$ i.e., of the type $ax^2 + c = 0$ **(Pure quadratic equation)**

(ii) $b \neq 0, c = 0$ i.e., of the type $ax^2 + bx = 0$

(iii) $b = 0$, $c = 0$ i.e., of the type $ax^2 = 0$

(iv) $b \neq 0, c \neq 0$ i.e., of the type $ax^2 + bx + c = 0$ **(Mixed or complete quadratic equation)**

Roots of quadratic equation : x = α is said to be root of the quadratic equation ax2 + bx + c = 0, a $\neq 0$ iff x = α satisfies the quadratic equation i.e. in other words the value of aα^2 + bα + c is zero.

Solving a quadratic equation : The determination of all the roots of a quadratic equation is called solving the quadratic equation.

Ex.1 Check whether the following are quadratic equations :

(i) $(x + 1)^2 = (x - 3)$ (ii) (x – 2) (x + 1) = (x – 1) (x + 3) (iii) (x – 3) (2x + 1) = x (x + 5)

Sol. (i) Here, the given equation is $(x + 1)^2 = 2(x - 3)$

$\Rightarrow$ $x^2+2x+1=2x-6 \Rightarrow x^2+2x-2x+1+6=0$

$\Rightarrow$ $x^2+7=0 \Rightarrow x^2+0.x+7=0$, which is of the form $ax^2+bx+c=0$

Hence, $(x + 1)^2 = 2(x - 3)$ is a quadratic equation.

(ii) Here, the given equation is (x – 2)(x + 3) = (x – 1) (x + 3)

$\Rightarrow$ $x^2+x-2x-2=x^2+3x-x-3 \Rightarrow x^2-x^2-x-2x-2+3=0 \Rightarrow -3+1=0$,

which is not of the form $ax^2+bx+c=0$

Hence, (x – 2) (x + 1) = (x – 1) is not a quadratic equation.

(iii) Here, the given equation is (x – 3) (2x + 1) x (x + 5)

$\Rightarrow$ $2x^2+x-6x-3=x^2+5x \Rightarrow 2x^2-x^2-5x-5x-3=0 \Rightarrow x^2-10x-3=0$,

which is of the form $ax^2+bx+c=0$

Hence, (x – 3) (2x + 1) = x(x + 5) is a quadratic equation.

Ex.2 In each of the following, determine whether the given values are the solution of the given equation or not :

(i) $\frac{2}{x^2}-\frac{5}{x}+2=0; x=5, x=\frac{1}{2}$ (ii) $a^2x^2-3abx+2b^2=0; x=\frac{a}{b}, x=\frac{b}{a}$

Sol. (i) Putting x = 5 and $x=\frac{1}{2}$ in the given equation.

$$\frac{2}{(5)^2}-\frac{5}{5}+2 \text{ and } \frac{2}{\left(\frac{1}{2}\right)^2}-\frac{5}{\left(\frac{1}{2}\right)}+2$$

$$\Rightarrow \frac{2}{25}-1+2 \text{ and } \frac{2}{\frac{1}{4}}-\frac{5}{\frac{1}{2}}+2 \quad \Rightarrow \quad \frac{2}{25}+1 \text{ and } 8-10+2 \Rightarrow \frac{27}{25} \text{ and } 0$$

i.e., x = 5 does not satisfy but $x=\frac{1}{2}$ satisfies the given equation.

Hence, x = 5 is not a solution but $x=\frac{1}{2}$ is a solution of $\frac{2}{x^2}-\frac{5}{x}+2=0$.

(ii) Putting $x=\frac{a}{b}$ and $x=\frac{b}{a}$ in the given equation.

$$a^2\left(\frac{a}{b}\right)^2-3ab\left(\frac{a}{b}\right)+2b^2 \text{ and } a^2\left(\frac{b}{a}\right)^2-3ab\left(\frac{b}{a}\right)+2b^2$$

$$\Rightarrow \frac{a^2}{b^2}+2b^2-3a^2 \text{ and } 0$$

i.e., $x=\frac{a}{b}$ does not satisfy but $x=\frac{b}{a}$ satisfies the given equation.

Hence, $x=\frac{b}{a}$ is a solution but $x=\frac{a}{b}$ is not a solution of $a^2x^2 - 3abx + 2b^2 = 0$.

Ex.3 Find the values of p and q for which $x = \frac{3}{4}$ and x = – 2 are the roots of the equation $px^2 + qx - 6 = 0$.

Sol. Since $x = \frac{3}{4}$ and x = – 2 are the roots of the equation $px^2 + qx - 6 = 0$.

$\therefore \quad p\left(\frac{3}{4}\right)^2 + q\left(\frac{3}{4}\right) - 6 = 0$ and $p(-2)^2 + q(-2) - 6 = 0$

$\Rightarrow \quad p \times \frac{9}{16} + q \times \frac{3}{4} - 6 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad \frac{9p + 12q - 96}{16} = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad 9p + 12q - 96 = 0$ and $4p - 2q - 6 = 0$

$\Rightarrow \quad 3p + 4q - 32 = 0$ …(i)

and $2p - q - 3 = 0$ …(ii)

Multiplying (2) by 4, we get $8p - 4q - 12 = 0$ …(iii)

Adding (1) and (3), we get p = 4

Putting the value of p in equation (2), we get

$$23 \times 4 - q - 3 = 0 \Rightarrow q = 5$$

Hence, p = 4, q = 5.

★ METHODS OF SOLVING QUADRATIC EQUATIONS

Solution by factorization method

Algorithm :

Stop-I : Factorize the constant term of the given quadratic equation.

Stop-II : Express the coefficient of middle term as the sum or difference of the factors obtained in step-I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Stop-III : Split the middle term in two parts obtained in stop-II

Stop-IV : Factorize the quadratic equation obtained in step-III by grouping method.

Ex.4 Solve the following quadratic equation by factorization method $x^2 - 2ax + a^2 - b^2 = 0$

Sol. Factors of the constant term $a^2 - b^2$ are(a – b)&(a + b) also coefficient of the middle term = – 2a = –[(a – b)+(a + b)]

$\Rightarrow \quad x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow \quad x^2 - \{(a - b) + (a + b)\} x + (a + b)(a - b) = 0$

$\Rightarrow \quad x^2 - (a - b) x - (a + b) x + (a - b)(a + b) = 0$

$\Rightarrow \quad x [x - (a - b)] - (a + b) [x - (a - b)] = 0$

$\Rightarrow \quad [x - (a - b)] [x - (a + b)] = 0$

$x - (a - b) = 0$ or $x - (a + b) = 0$

$x = a - b, x = a + b$

Ex.5 Solve the quadratic equation $5x^2 = -16x - 12$ by factorization method.

Sol. $5x^2 = -16x - 12$

$5x^2 + 16x + 12 = 0$

$5x^2 + 10x + 6x + 12 = 0$

$5x(x + 2) + 6(x + 2) = 0$

$(x + 2)(5x + 6) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$5x + 6 = 0 \Rightarrow x = \frac{-6}{5}$

Solution by factorization method
Algorithm :

Stop-I : Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Stop-II : Make the coefficient of x^2 unite by dividing throughout by it, if it is not unity that is obtain

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Stop-III : Shift the coefficient term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a} = -\frac{c}{a}$

Stop-IV : Add square of half of the coefficient of x. i.e., $\left(\frac{b}{2a}\right)^2$ on both sides to obtain.

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Stop-V : Write L.H.S. as the perfect square and simplify R.H.S. to get $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Stop-VI : Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Stop-VII : Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on R.H.S. i.e., $x = -\frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Ex.6 Solve : $9x^2 - 15x + 6 = 0$

Sol. Here, $9x^2 - 15x + 6 = 0$

$\Rightarrow x^2 - \frac{15}{9}x + \frac{6}{9} = 0$ [Dividing throughout by 9]

$\Rightarrow x^2 - \frac{5}{3}x + \frac{2}{3} = 0$

$\Rightarrow x^2 - \frac{5}{3}x - \frac{2}{3} = 0$ [Shifting the constant term on RHS]

$\Rightarrow x^2 - 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{2}{3}$ [Adding square of half of coefficient x on both sides]

$\Rightarrow \left(x - \frac{5}{6}\right)^2 = \frac{25}{36} - \frac{2}{3} \Rightarrow \left(x - \frac{5}{6}\right)^2 = \frac{25 - 24}{36} \Rightarrow \left(x - \frac{5}{6}\right)^2 = \frac{1}{36}$

$\Rightarrow x - \frac{5}{6} = \pm\frac{1}{6}$ [Taking square root of both sides]

$\Rightarrow x = \frac{5}{6} = \pm\frac{1}{6} \Rightarrow x = \frac{5}{6} + \frac{1}{6} = 1$ or, $x = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow x = 1$ or, $x = \frac{2}{3}$

Ex.7 Solve the equation $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$ by the method of completing the square.

Sol. We have,

$$x^2-(\sqrt{3}+1)x+\sqrt{3}=0 \quad \Rightarrow \quad x^2-(\sqrt{3}+1)x=-\sqrt{3}$$

$$\Rightarrow \quad x^2-2\left(\frac{\sqrt{3}+1}{2}\right)x+\left(\frac{\sqrt{3}+1}{2}\right)^2=-\sqrt{3}+\left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{3}+1}{2}\right)^2=\frac{-4\sqrt{3}+(\sqrt{3}+1)^2}{4}$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{3}+1}{2}\right)^2=\left(\frac{\sqrt{3}-1}{2}\right)^2 \quad \Rightarrow x-\frac{\sqrt{3}+1}{2}=\pm\frac{\sqrt{3}-1}{2}$$

$$\Rightarrow \quad x-\frac{\sqrt{3}+1}{2}\pm\frac{\sqrt{3}-1}{2} \quad \Rightarrow x=\sqrt{3},\ 1$$

Hence, the roots are $\sqrt{3}$ and 1.

Solution by Quadratic Formula "Sreedharacharya's Rule"

Consider quadratic equation $ax^2+bx+c=0, a\neq 0$ then $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore$ The roots of x are

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x=\frac{-b+\sqrt{D}}{2a} \text{ or, } x=\frac{-b-\sqrt{D}}{2a}$$, where D = b^2 – 4ac

Thus, if $D=b^2-4ac\geq 0$, then the quadratic equation ax2 + bx + c = 0 has real roots α and β given by

$$\alpha=\frac{-b+\sqrt{D}}{2a} \text{ and } \beta=\frac{-b-\sqrt{D}}{2a}$$

Discriminate : If $ax^2+bx+c=0, a\neq 0 (a,b,c\in R)$ is a quadratic equation, then the expression b^2 – 4ac is known as its discriminate and is generally denoted by D or Δ.

Ex.8 Solve the quadratic equation x^2 – 6x + 4 = 0 by using quadratic formula (Sreedharacharya's Rule).

Sol. On comparing the given equation x^2 – 6x + 4 = 0 with the standard quadratic equation ax^2 + bx + c = 0, we get a = 1, b = – 6, c = 4

Hence the required roots are

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(4)}}{2(1)}=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm\sqrt{20}}{2}=\frac{6\pm\sqrt{4\times 5}}{2}$$

$$=\frac{2(3\pm\sqrt{5})}{2}=3\pm\sqrt{5}$$

COMPETITION WINDOW

SOLUTIONS OF EQUATIONS REDUCIBLE TO QUADRATIC FORM

Equations which are not quadratic at a glance but can be reduced to quadratic equations by suitable transformations Some of the common types are :

Type-I : $ax^4 + bx^2 + c = 0$

This can be reduced to a quadratic equation by substituting $x^2 = y$ i.e., $ay^2 + by + c = 0$

e.g. Solve $2x^4 - 5x^2 + 3 = 0$

Putting $x^2 = y$, we get $2y^2 - 5y + 3 = 0$

$\Rightarrow \quad (2y-3)(y-1)=0 \Rightarrow y=\frac{3}{2}$ or 1

$\Rightarrow \quad (2y-3)(y-1)=0 \Rightarrow y=\frac{3}{2}$ or 1

Type-II : $ai p(x)\}^2 + b.p(x) + c = 0$ where p(x) is an expression in 'x'

Put $p(x) = y$, $\{p(x)\}^2 = y^2$ to get the quadratic equation $ay^2 + by + c = 0$.

e.g. Solve $(x^2 + 3x)^2 - (x^2 + 3x) - 6 = 0$, $x \in R$

Putting $x^2 + 3x = y$, we get $y2 - y - 6 = 0$

Solving, we get $y = 3$ or -2

$\Rightarrow \quad x^2+3x=3$ or $x^2+3x=-2 \qquad \Rightarrow \quad x\frac{-3\pm\sqrt{21}}{2}$ or $x=-2$ or -1.

Type-III : ap$(x)+\frac{b}{p(x)}=c$, where p(x) is an expression in x.

Put $p(x) = y$ to obtain the quadratic equation $ay^2 - cy + b = 0$.

e.g. Solve $\frac{x}{x+1}+\frac{x+1}{x}=\frac{34}{15}$

Putting $\frac{x}{x+1}=y$, we get, $y=\frac{1}{y}=\frac{34}{15} \qquad \Rightarrow \quad 15y^2-34y+15=0 \Rightarrow y=\frac{5}{3}$ or $\frac{3}{5}$

$\Rightarrow \quad \frac{x}{x+1}=\frac{5}{3}$ or $\frac{x}{x+1}=\frac{3}{5} \Rightarrow x=\frac{-5}{2}$ or $\frac{3}{2}$

Type-IV : (i) $a\left[x^2+\frac{1}{x^2}\right]+b\left[x+\frac{1}{x}\right]+c=0$ **(ii)** $a\left[x^2+\frac{1}{x^2}\right]+b\left[x-\frac{1}{x}\right]+c=0$

If the coefficient of b in the given equation contains $x+\frac{1}{x}$, then replace $x^2+\frac{1}{x^2}$ by $\left(x^2+\frac{1}{x^2}\right)^2-2$ and put $x+\frac{1}{x}=y$. In case the coefficient of b is $x-\frac{1}{x}$, then replace $x^2+\frac{1}{x^2}$ by $\left(x-\frac{1}{x}\right)^2+2$ and put $x-\frac{1}{x}=y$.

e.g. Solve $9\left[x^2+\frac{1}{x^2}\right]-9\left[x+\frac{1}{x}\right]-52=0$

Putting $x+\frac{1}{x}=y$, we get : $9(y^2 - 2) - 9y - 52 = 0$

$\Rightarrow \quad y=\frac{10}{3}$ or $y=-\frac{7}{3} \qquad \Rightarrow x+\frac{1}{x}=\frac{10}{3}$ or $x+\frac{1}{x}=-\frac{7}{3}$

$\Rightarrow \quad x=\frac{1}{3}$ or 3 or $x=\frac{-7\pm\sqrt{13}}{6}$

Type-V : (x + a) (x + b) (x + c) (x + d) + k = 0, such that a + b = c + d.

Rewrite the equation in the form

{(x + a) (x + b)} . {(x + c) (x + d)} + k = 0

Put $x^2 + x(a + b) = x^2 + x(c + d) = y$ to obtain a quadratic equation in y i.e. (y + ab) (y + cd) = k.

e.g. Solve (x + 1) (x + 2) (x + 3) (x + 4) = 120

$\because$ 1 + 4 = 2 + 3, we write the equation in the following form :

{(x + 1) (x + 4)} . {(x + 2) (x + 3)} = 120

$\Rightarrow$ $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Putting $x^2 + 5x = y$, we get (y + 4) (y + 6) = 120

$\Rightarrow$ y = – 16 or 6 $\Rightarrow$ $x^2 + 5x = -16$ or $x^2 + 5x = 6$

$\Rightarrow$ x = – 6 or 1 ($x^2 + 5x + 16$ has no real solution)

Type-VI : $\sqrt{ax+b} = (cx+d)$

Square both sides to obtain $(ax + b) = (cx + d)^2$

or $c^2x^2 + (2cd - a)x + d^2 - b = 0$

Reject those values of x, which do not satisfy both $ax+b \geq 0$ and $cx+d \geq 0$

e.g. Solve : $\sqrt{2x+9} + x = 13$

$\Rightarrow$ $(2x + 9) = (13 - x)^2$ (on squaring both sides)

$\Rightarrow$ $x^2 - 28x + 160 = 0$

$\Rightarrow$ x = 20 or 8

x = 20 does not satisfy $2x + 9 \geq 0$. So, x = 8 is the only root.

Type-VII : $\sqrt{ax^2 + bx + c} = dx + e$

Square both sides to obtain the quadratic equation $x^2(a - d^2) + x(b - 2de) + (c - e^2) = 0$. solve it and reject those value of x which do not satisfy ax2 + bx + c $\geq$ 0 and dx + e $\geq$ 0.

e.g. Solve : $\sqrt{3x^2 + x + 5} = x - 3$

$\Rightarrow$ $3x^2 + x + 5 = (x - 3)^2$ (On squaring both sides)

$\Rightarrow$ $2x^2 + 7x - 4 = 0 \Rightarrow x = \frac{1}{2}$ or -4

No value of x satisfy $3x^2 + x + 5 \geq 0$ and $x - 3 \geq 0$

Type-VIII : $\sqrt{ax+b} \pm \sqrt{cx+d} = e$

Square both sides and simplify in such a manner that the expression involving radical sing on one side and all other terms are on the other side. square both sides of the equation thus obtained and simplify it to obtain a quadratic in x. Reject these values which do not satisfy ax + b $\geq$ 0 and cx + d $\geq$ 0.

e.g. Solve : $\sqrt{4-x} + \sqrt{x+9} = 5$

$\Rightarrow$ $\sqrt{4-x} = 5 - \sqrt{x+9}$

$\Rightarrow$ $x + 15 = 5\sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ $(x+15)^2 = 25\sqrt{x+9}$ (on squaring both sides)

$\Rightarrow$ x = 0 or – 5

Clearly, x = 0 and x = – 5 satisfy 4 – x $\geq$ 0 and x + 9 $\geq$ 0.

Hence, the roots are 0 and – 5

★ NATURE OF THE ROOTS OF THE QUADRATIC EQUATION

Let the quadratic equation be ax2 + bx + c = 0. …(i)
Where a $\neq$ b and a, b, c $\in$ R.

The roots of the given equation are given by $x = \frac{-b \pm \sqrt{D}}{2a}$.

i.e., of α and β are two roots of the quadratic equation (i). Them.

$$\alpha = \frac{-b+\sqrt{D}}{2a} \text{ and } \beta = \frac{-b-\sqrt{D}}{2a}$$

Now, the following cases are pollible.

Case-I : When D > 0.
Roots are real and unequal (distinct).
The roots are given by $\alpha = \frac{-b+\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$

Remark : Consider a quadratic equation ax2 + bx+ c = 0. where a,b,c$\in$Q, a $\neq$ b and D > 0 them :

(i) If D is a perfect square, then roots are rational and unequal.

(ii) If D is not a perfect square, then roots are irrational and unequal. If one root is of the form $p+\sqrt{q}$ (where p is rational and $\sqrt{q}$ is a surd) then the other root will be $p-\sqrt{q}$.

Case-II : When D = 0.
Roots are real and equal and each root $\alpha = \frac{-b}{2a} = \beta$

Case-III : When D < 0.
No real roots exist. Both the roots are imaginary.

Remark : If D < 0, the roots are of the form a $\pm$ ib (a,b$\in$R&I = $\sqrt{-1}$). If one root is a + ib, then other root will be a – ib.

e.b. $x^2 - 3x + 12 = 0$ has D = – 39 < 0

$\therefore$ It's roots are, $\alpha = \frac{-b+\sqrt{D}}{2a}$ and $\beta = \frac{-b-\sqrt{D}}{2a}$

or $\alpha = \frac{3+\sqrt{-39}}{2}$ and $\beta = \frac{3-\sqrt{-39}}{2}$

or $\alpha = \frac{3}{2} + \frac{i\sqrt{39}}{2}$ and $\beta = \frac{3}{2} - \frac{i\sqrt{39}}{2}$

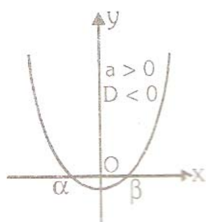
COMPETITION WINDOW

GEOMETRICAL REPRESENTATION OF QUADRATIC EXPRESSION

Consider the quadratic expression, y = ax2 + bx + c, a $\neq 0$ & a,b,c $\in$ R then :

(i) The graph between x,y is always a parabola. If a > 0, then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.

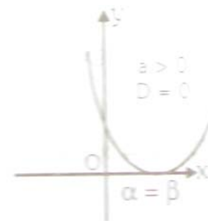
(ii) The graph of $y = ax^2 + bx + c$ can be divided into 6 categories which are as follows :
(Let the roots of the equation $ax^2 + bx + c = 0$ be α and β)



Fig(i)
Roots are real
and distinct

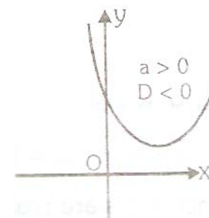
$ax^2 + bx + c > 0 \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$

$ax^2 + bx + c < 0 \forall x \in (\alpha, \beta)$



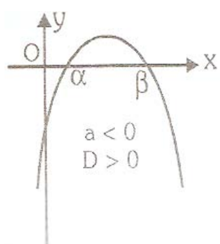
Fig(i)
Roots are
coincident

$ax^2 + bx + c > 0 \forall x \in R - (\alpha)$



Fig(iii)
Roots are
complex conjugates

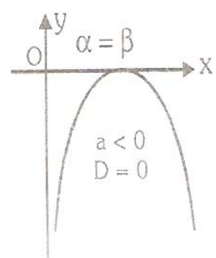
$ax^2 + bx + c > 0 \forall x \in R$



Fig(iv)
Roots are real
and distinct

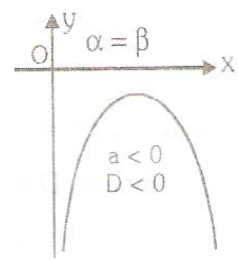
$ax^2 + bx + c < 0 \forall x \in (\alpha, \beta)$

$ax^2 + bx + c > 0 \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$



Fig(v)
Roots are
coincident

$ax^2 + bx + c > 0 \forall x \in R - (\alpha)$



Fig(vi)
Roots are
complex conjugates

$ax^2 + bx + c > 0 \forall x \in R$

Remark : (i) The quadratic expression $ax^2 + bx + c > 0\ \forall\ x \in R \Rightarrow a > 0, D < 0$ (fig (iii))

(ii) The quadratic expression $ax^2 + bx + c > 0\ \forall\ x \in R \Rightarrow a > 0, D < 0$ (fig (vi))

Ex.9 Find the nature of the roots of the following equations. If the real roots exist, find them.

(i) $2x^2 - 6x + 3 = 0$ (ii) $2x^2 - 3x + 5 = 0$

Sol. (i) The given equation $2x^2 - 6x + 3 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

a = 2, b = – 6 and c = 3.

$\therefore$ Discriminant, $D = b^2 - 4ac = (-6)^2 - 4.2.3 = 36 - 24 = 12 > 0$

$\because$ D > 0, roots are real and unequal.

Now, by quadratic formula, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{6 \pm \sqrt{12}}{2 \times 2} = \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{3 \pm \sqrt{3}}{2}$

Hence the roots are $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

(ii) Here, the given equation is $2x^2 - 3x + 5 = 0$;
Comparing it with $ax^2 + bx + c = 0$, we get
a = 2, b = – 3 and c = 5.
∴ Discriminant, $D = b^2 - 4ac = 9 - 4\times2\times5 = 9 - 40 = -31$
∵ D < 0, the equation has no real roots.

Ex.10 Find the value of k for each of the following quadratic equations, so that they have real and equal roots :
(i) $9x^2 + 18kx + 16 = 0$ (ii) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

Sol. (i) The given equation $9x^2 + 8kx + 16 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
a = 9, b = 8k and c = 16.
∴ Discriminant, $D = b^2 - 4ac = (8k)^2 - 4\times9\times16 = 64k^2 - 576$
Since roots are real and equal, so
$D = 0 \Rightarrow 64k^2 - 576 = 0 \Rightarrow 64k^2 = 576$
$\Rightarrow k^2 = \frac{576}{64} = - \Rightarrow k = \pm3$
Hence, k = 3, – 3

(ii) The given equation$(k + 1)x^2 - 2(k - 1)x + 1 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
a = (k + 1), b – 2 (k – 1) and c = 1
∴ Discriminate, $D = b^2 - 4ac = 4(k - 1)^2 - 4(k + 1)\times1$
= $4(k^2 - 2k + 1) - 4k - 4$
$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 4k^2 - 12k$
Since roots are real and equal, so
$D = 0 \Rightarrow 4k^2 - 12k = 0 \Rightarrow 4k(k - 3) = 0$
$\Rightarrow$ either k = 0 or k – 3 = 0 $\Rightarrow$ k = 0 or k = 3
Hence, k = 0, 3.

Ex.11 Find the set of value of k for which the equations $kx^2 + 2x + 1$ has distinct real roots.

Sol. The given equation is $kx^2 + 2x + 1 = 0$
∴ $D = (4 - 4 \times k\times1) = 4c - 4k$
For distinct and real roots, we must have, D > 0.
Now, $D = (4 - 4k) > 0 \Leftrightarrow 4 > 4k \Leftrightarrow 4k < 4 \Leftrightarrow k < 1$.
∴ Required set = $\{k \in R : k < 1\}$

Ex.12 Find the of k for which the equations $5x^2 - kx + 4 = 0$ has real roots.

Sol. The given equation is $5x^2 - kx + 4 = 0$
∴ $D = k^2 - 4\times5\times4 = k^2 - 80$
For real roots, we must have, $D \geq 0$.
Now, $D > 0 \Leftrightarrow k^2 - 80 \geq 0 \Leftrightarrow k^2 \geq 80 \Leftrightarrow k \geq \sqrt{80}$ or $k \leq -\sqrt{80} \Leftrightarrow k \geq 4\sqrt{5}$ or $k \leq -4\sqrt{5}$.

COMPETITION WINDOW

ROOTS UNDER PARTICUALR CASES

(A) Let the quadratic equation $ax^2 + bx + c = 0$ has real roots and

(i) If b = 0 $\Leftrightarrow$ roots are of equal magnitude but of opposite sign.

(ii) If c = 0 $\Leftrightarrow$ one roots is zero and the other is $-\frac{b}{a}$

(iii) If a = c $\Leftrightarrow$ roots are of opposite sign.

(iv) If a > 0 c < 0, a < 0 c > 0 $\Leftrightarrow$ roots are of opposite sign.

(v) If a > 0, b > 0, c < 0, a < 0, b < 0, c < 0 $\Leftrightarrow$ both roots are negative $(\alpha+\beta<0\,\&\,\alpha\beta>0)$

(vi) If a > 0, b < 0, c > 0, a < 0, b > 0, c < 0 $\Leftrightarrow$ both roots are positive $(\alpha+\beta<0\,\&\,\alpha\beta>0)$

(vii) If a + b + c = 0 $\Leftrightarrow$ One of the roots is 1 and the other roots is $\frac{c}{a}$.

(viii) If a = 1 b, c $\in$ Z and the roots are rational numbers, then these roots must be integers.

(ix) If a, b, c $\in$ Q and D is a perfect square $\Leftrightarrow$ roots are rational.

(x) (A) If a, b, c $\in$ Q and D is positive but not a perfect square $\Leftrightarrow$ roots are irrational.
(B) If $ax^2 + bx + c = 0$ is satisfied by more then two values, it is an identity and a = b = c = 0 and vice versa
(C) The quadratic equation whose roots are reciprocal of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (i.e. the coefficients are writer in reverse order).

★ SUM & PRODUCT OF THE ROOTS

Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

Then $\alpha = \frac{-b+\sqrt{+b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{+b^2-4ac}}{2a}$

$\therefore$ The sum of roots $\alpha+\beta = -\frac{b}{a} = -\frac{Ceoff.of\ x}{Ceoff.of\ x^2}$

and product of roots = $\alpha.\beta = \frac{c}{a} = -\frac{\cos\tan t\,term}{coefficient\,of\ x^2}$

★ FORMATION OF QUADRATIC EQUATION

Consider the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.
Let α and β be the roots of the quadratic equation

$\therefore$ $\alpha+\beta = -\frac{b}{a}$ and $\alpha.\beta = \frac{c}{a}$

Hence the quadratic equation whose roots are α and β is given by

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

i.e. x^2 – (sum of the roots) x + product of the roots = 0

Ex.13 Form the quadratic equation in each of the following cases when the roots are :

(i) $2+\sqrt{5}2-\sqrt{5}$ (ii) a and $\frac{1}{a}$

Sol. (i) Here roots are $\alpha = 2+\sqrt{5}$ and $\beta = 2-\sqrt{5}$

∴ Sum of roots $= \alpha+\beta(2+\sqrt{5})+(2-\sqrt{5})$

∴ $\alpha+\beta = 4$

and product of the roots $= \alpha.\beta = (2+\sqrt{5}).(2-\sqrt{5}) = 4-5 = -1$

∴ $\alpha\,\beta = -1$

∴ Required equation is
x^2 – (sum of the roots) x + product of the roots = 0

or $x^2-(\alpha+\beta)x+\alpha\beta = 0$

or $x^2-(4)x+(-1) = 0$

∴ $x^2-4x-1 = 0$

(ii) Here roots are a and $\frac{1}{a}$

∴ $\alpha+\beta = a+\frac{1}{a}$ and $\alpha.\beta = a\times\frac{1}{a} = 1$

Here the required equation is $x^2-\left(a+\frac{1}{a}\right)x+1 = 0$

COMPETITION WINDOW

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE A COMMON ROOT

Suppose that the quadratic equation $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ (where a, a' ≠ 0 and ab' – a'b ≠ 0) have a common root. Let this common root be α Than $a\alpha^2 b\alpha + c = 0$ and $a'\alpha^2 b'\alpha + c' = 0$

Solving the above equations, we get,

$$\frac{\alpha^2}{bc'-b'c} = \frac{\alpha}{a'c-ac'} = \frac{1}{ab'-a'b} \quad\Rightarrow\quad \alpha^2 = \frac{bc'-b'c}{ab'-a'b} \text{ and } \alpha = \frac{a'c-ac'}{ab'-a'b}$$

Eliminating α, we get : $\frac{(a'c-ac')^2}{(ab-a'b)^2} = \frac{bc'-b'c}{ab'-a'b} \quad\Rightarrow\quad (a'c-ac')^2 = (bc'-b'c)(ab'-a'b)$

This is the required condition for two quadratic equations to have a common root. To obtain the common root, make coefficient of x^2 in both the equation same and subtract one equation from the other to obtain a linear equation in x. Solve it for x to obtain the common root.

Ex. For which value of k will the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root.

Sol. Let the common root be α than, $\alpha^2-k\alpha-21 = 0$ and $\alpha^2-3k\alpha-35 = 0$.

Solving by Cramer rule, we have : $\frac{\alpha^2}{-35k-63k} = \frac{\alpha}{-21-35k} = \frac{1}{-3k+k}$

∴ $\alpha = \frac{-98k}{-56} = \frac{-7k}{4}$ and $\frac{7k}{4} = \frac{28}{k} \Rightarrow 7k^2 = 28\times4 \Rightarrow k = \pm4$

CONDITION FOR TWO QUADRATIC EQUATION TO HAVE THE SAME ROOT

Two quadratic equations $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$ have the same roots if and only if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

★ **APPLICATIONS OF QUADRATIC EQUATIONS**

Algorithm :

The method of problem solving consists of the following three steps :-

Stop-I : Translating the word problem in to symbolic language (mathematical statement) which means identifying relationships existing in the problem & then forming the quadratic equation.

Stop-II : Solving the quadratic equation thus formed

Stop-III: Interpreting the solution of the equation which means translating the result of mathematical statement into verbal language.

Type-I : Problems Based On Numbers.

Ex.14 The difference of two numbers is 3 and their product is 504. Find the numbers.

Sol. Let the required numbers be x and (x – 3). Then,

$x(x-3) = 504$

$\Rightarrow x^2 - 3x - 504 = 0 \Rightarrow x^2 - 24x + 21x - 504 = 0$

$\Rightarrow x(x-24) + 21(x-24) = 0 \Rightarrow (x-24)(x+21) = 0$

$\Rightarrow x - 24 = 0$ or $x + 21 = 0 \Rightarrow x = 24$ or $x = -21$

If x = – 21, then the numbers are – 21 and – 24.

Again, if x = 24, then the numbers are 24 and 21.

Hence, the numbers are – 21, – 24 or 24, 21

Ex.15 The sum of the square of two consecutive odd positive integers is 290. find the integers.

Sol. Let the two consecutive odd positive integers be x and (x + 2). Then.

$x^2 + (x+2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x = 290 \Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0 \Rightarrow x(x+13) - 11(x+13) = 0$

$\Rightarrow (x+13)(x-11) = 0 \Rightarrow x = -13$ or $x = 11$

If x = – 21, then the numbers are – 21 and – 24.

But – 13, is not an odd positive integer.

Hence, the required integers are 11 and 13.

Type-II : Problems Based On Ages :

Ex.16 Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two fifth of Varun's age. Find their present ages.

Sol. Let the present ages of Varun and Swati be x years and y years respectively.

Seven years ago,

Varun's age = (x – 7) years and Swati's age = (y – 7) years.

$\therefore (x-7) = 5(y-7)^2 \Rightarrow x - 7 = 5(y^2 - 14y + 49)$

$\Rightarrow x = 5y^2 - 70y + 245 + 7 \Rightarrow x = 5y^2 - 70y + 252$ …(i)

Three years hence,

Varun's age = (x + 3) years and Swati's age = (y + 3) years.

$\therefore (y+3) = \frac{2}{5}(x+3) \Rightarrow 5y + 15 = 2x + 6 \Rightarrow x = \frac{5y+9}{2}$ …(ii)

From (i) and (ii) we get $5y^2 - 70y + 252 = \frac{5y+9}{2}$

$\Rightarrow 10y^2 - 140y + 504 = 5y + 9 \Rightarrow 10y^2 - 145y + 495 = 0 \Rightarrow 2y^2 - 29y + 99 = 0$

$\Rightarrow 2y^2 - 18y - 11y + 99 = 0 \Rightarrow 2y(y-9) - 11(y-9) = 0 \Rightarrow (y-9)(2y-11) = 0 \Rightarrow y = 9$ or $y = \frac{11}{2}$

$\because y = \frac{11}{2}$ is not possible $\left[\because \frac{11}{2} < 7\right]$ So, y = 9.

$\therefore \frac{5\times 9+9}{2} = 27$ [From (ii)]

Hence, the Varu's present age is 27 years and Swati's present age is 9 years..

Type-III : Problems Based On Geometrical Concepts :

Ex.17 The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol. Let ΔABC be a right triangle, right angled at B.

Let AB = x. Then

AC = (2x + 1) and BC = (2x + 1) – 2 = 2x – 1

$\Rightarrow$ ΔABC, $AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow$ $(2x + 1)^2 = x^2 + (2x - 1) \Rightarrow 4x^2 + 4x + 1 = x^2 + 4x^2 - 4x + 1$

$\Rightarrow$ $x^2 = 8x \Rightarrow x = 8$ cm,

$\therefore$ BC = 2x – 1 = $2\times8 + 1$ = 15 cm

AC = 2x + 1 = $2\times8 + 1$ = 17 cm

Hence, the sides of the given triangle are 8cm, 15 cm and 17 cm.

Type-IV : Problems Based On Perimeter/Age :

Ex.18 Is it possible to design a rectangular park of perimeter 80 cm and area 400 m^2? If so, find its length and breadth.

Sol. Let the length and breadth of the rectangular park be ℓ and b respectively. Then,

$2(\ell + b) = 80$

$\ell + b = 40 \Rightarrow \ell = (40 - b)$

And area of the park = 400 m^2

$\therefore \ell b = 400$

$\Rightarrow$ $(40 - b)\, b = 400 \Rightarrow 40b - b^2 = 400$

$\Rightarrow$ $b^2 - 40b + 400 = 0 \Rightarrow b^2 - 20b + 400 = 0$

$\Rightarrow$ $b(b - 20) - 20(b - 20) = 0 \Rightarrow (b - 20)(b - 20) = 0$

$\Rightarrow$ $(b - 20)^2 = 0 \Rightarrow b - 20 \Rightarrow b = 20$ m

$\therefore$ $\ell = 40 - b = 40 - 20 = 20$ m

Hence, length and breadth of the park are 20 m and 20 m respectively.

Thus, it is possible to design a rectangular park of perimeter 80 m and area 400 m^2

Type-V : Problems Based On Time and Distance :

Ex.19 A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less tor the same journey. Find the speed of the train.

Sol. Let the speed of the train be x km/h. Then,

Time taken to cover the distance of 360 km = $\frac{360}{x}$ hours.

If the speed of the train increased by 5 km/h. Them,

Time taken to cover the same distance = $\left(\frac{360}{x+5}\right)$h

According to the question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow$ $\frac{360(x+5) - 360x}{x(x+5)} = 0 \Rightarrow 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow$ $x^2 + 5x - 1800 = 0 \Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow$ $x(x+45) - 40(x+45) = 0 \Rightarrow (x+45)(x-40) = 0$

$\Rightarrow$ $x = -45$ or $x = 40$

But the speed can not be negative.

Hence, the speed of the train is 40 km/h.

Type-VI : Problems Based On Time and Work :

Ex.20 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank respectively. Find the time in which each tap can separately fill the tank.

Sol. Let the tap of larger diameter takes x hours to fill the tank. Then, the tap of smaller diameter takes (x + 10) hours to fill the tank.

∴ The portion of tank filled by the larger tap in one hour $=\frac{1}{x}$, the portion of tank filled by the smaller tap in one hour $=\frac{1}{x+10}$

And the portion of tank filled by both the smaller and the larger tap in one hour $=\frac{1}{9\frac{3}{8}}=\frac{8}{75}$

∴ $\frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$

⇒ $\frac{x+10+x}{x(x+10)}=\frac{8}{75} \Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$

⇒ $15x + 750 = 8x^2 + 80x \Rightarrow 8x^2 - 70x - 750 = 0$

⇒ $4x^2 - 35x - 375 = 0 \Rightarrow 4x^2 - 60x + 25x - 375 = 0$

⇒ $4x(x - 15) + 25(x - 15) = 0 \Rightarrow (x - 15)(4x + 25) = 0$

⇒ $x = 15$ or $x=\frac{-25}{4}$

But the speed can not be negative.

Hence, the larger tap takes 15 hours and the smaller tap takes 25 hours.

Type-VI : Miscellaneous Problems :

Ex.21 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol. Let the number of students be x. Then,

The number of apples received by each student = $\frac{300}{x}$

if there is 10 more students, i.e., (x + 10) students. Then,

The number of apples received by each student = $\frac{300}{x+10}$

According to the question, $\frac{300}{x}-\frac{300}{x+10}=1$

⇒ $\frac{300x+3000-300x}{x(x+10)}=1 \Rightarrow 3000=x^2+10x$

⇒ $x^2+10x-3000=0 \Rightarrow x^2+60x-50x-3000=0$

⇒ $x(x+60)-50(x+60)=0 \Rightarrow (x+60)(x-50)=0$

⇒ $x=-60$ or $x=50$

But the number of students can not be negative.

Hence, the number of students is 50.

SYNOPSIS

Quadratic Equation : A quadratic equation in one variable x is of the form $ax^2 + bx + c = 0,\ a \neq 0$ where a, b and c are real numbers.

Roots of the quadratic equation : A real number α is said to be a root of the quadratic equation or a zero of the quadratic polynomial if and only if α satisfies the equation i.e., which make LHS = RHS.

Sreedharacharya formula : $ax^2 + bx + c = 0,\ a \neq 0,\ b^2 - 4ac \geq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \alpha \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Nature of roots : A quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ has :

(i) No real roots if D < 0. (ii) Two distinct real roots if D > 0. (iii) Two equal real roots if D = 0.

Relation between roots of equation : $ax^2 + bx + c = 0,\ a \neq 0$

Sum of roots = $\alpha + \beta = \frac{-b}{a}$, Product of roots = $\alpha\beta = \frac{c}{a}$

Formation of quadratic equation when roots are given : $ax^2 + bx + c = 0$ $[x^2 - (\alpha + \beta)x + \alpha\beta]$

EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

OBJECTIVE TYPE QUESTIONS

CHOSSE THE CORRECT ONE

1. Which of the following quadratic expression can be expressed as a product of real linear factors?

(A) $x^2 - 2x + 3$ (B) $3x^2 - \sqrt{2x} - \sqrt{3}$ (C) $\sqrt{2x^2} - \sqrt{5x} + 3$ (D) None of these

2. Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6. The other starts with a wrong value of q and finds the roots to be 2 and – 9. Find the correct roots of the equation :

(A) 3, 4 (B) – 3, – 4 (C) 3, – 4 (D) – 3, 4

3. Solve for x : $15x^2 - 7x - 36 = 0$

(A) $\frac{5}{9}, -\frac{4}{3}$ (B) $\frac{9}{5}, -\frac{4}{3}$ (C) $\frac{5}{9}, -\frac{3}{4}$ (D) None of these

4. Solve for y : $7y^2 - 6y - 13\sqrt{7} = 0$

(A) $\sqrt{7}, 2\sqrt{7}$ (B) $3, \frac{2}{\sqrt{7}}$ (C) $\frac{13}{\sqrt{7}}, -\sqrt{7}$ (D) None of these

5. Solve for x : $6x^2 + 40x = 31$

(A) $\frac{3}{8}, \frac{2}{5}$ (B) $\frac{3}{8}, \frac{3}{2}$ (C) $0, \frac{8}{3}$ (D) $\frac{8}{3}, \frac{5}{2}$

6. Determine k such that the quadratic equation $x^2 + 7(3 + 2k) - 2x(1 + 3k) = 0$ has equal roots :

(A) 2, 7 (B) 7, 5 (C) $2, -\frac{10}{9}$ (D) None of these

7. Discriminant of the roots of the equation $-3x^2 + 2x - 8 = 0$ is

(A) – 92 (B) – 29 (C) 39 (D) 49

8. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is

(A) No real roots (B) 1 real root (C) Can't be determined (D) None of these

9. The roots of $a^2x^2 + abx = b^2$, $a \neq 0$ are :

(A) Equal (B) Non-real (C) Unequal (D) None of these

10. The equation $x^2 - px + q = 0$ p, q $\in$ R has no real roots if :

(A) $p^2 > 4q$ (B) $p^2 < 4q$ (C) $p^2 = 4q$ (D) None of these

11. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :

(A) $\pm\left[\frac{2}{3}\right]$ (B) $\pm\left[\frac{4}{3}\right]$ (C) ± 4 (D) ± 6

12. Find the value of k such that the sum of the square of the roots of the quadratic equation $x^2 - 8x + k = 0$ is 40 :

(A) 12 (B) 2 (C) 5 (D) 8

13. Find the value of p for which the quadratic equation $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots :

(A) $-1, \frac{2}{3}$ (B) 3, 5 (C) $1, -\frac{4}{3}$ (D) $\frac{4}{3}, 2$

14. The length of a hypotenuse of a right triangle exceeds the length of its base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle (in cm) :

(A) 6, 8 10 (B) 7, 24, 25 (C) 8, 15, 17 (D) 7, 40, 41

15. A two digit number is such that the product of it's digits is 12. When 9 is added to the number, the digits interchange their places, find the number :

(A) 62 (B) 34 (C) 26 (D) 43

16. A plane left 40 minutes late due to bad weather and in order to reach it's destination, 1600 km away in time, it had to increase it's speed by 400 km/h from it's usual speed. Find the usual speed of the plane :

(A) 600 km/h (B) 750 km/h (C) 800 km/h (D) None of these

17. The sum of the squares of two consecutive positive odd numbers is 290. Find the sum of the numbers :

18. A shopkeeper buys a number of books for Rs. 80. If he had bought 4 more for the same amount, each book would have cost Re. 1 less. How many books did he buy?

(A) 8 (B) 36 (C) 24 (D) 28

19. The squares have sides x cm and (x + 4) cm. The sum of their areas is 656 cm^2. find the sides of the square.

(A) 8 cm, 12 cm (B) 12 cm, 15 cm (C) 6 cm, 10 cm (D) 16 cm, 20 cm

20. The real values of a for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite signs are given by :

(A) $a > 6$ (B) $a > 9$ (C) $0 < a < 4$ (D) $a < 0$

OBJECTIVE					ANSPER KEY									EXERCISE -I
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	B	B	B	C	D	C	A	A	C	B	B	A	A	C	B
Que.	**16**	**17**	**18**	**19**	**20**										
Ans.	C	B	B	D	C										

EXERCISE – 2 (FOR SCHOOL/BOARD EXAMS)

SUBJECTIVE TYPE QUESTIONS

VERY SHORT ANSWER TYPE QUESTIONS

1. State which of the following equations are quadratic equation :

(i) $3x+\frac{1}{x}-8=0$ (ii) $18x^2-6x=0$ (iii) $x^2-5x=7-6x^3$ (iv) $x^2=25$ (v) $6x^5+3x^2-7=0$

(vi) $x+\frac{1}{x^2}=3$ (vii) $5x^2+6x=7$ (viii) $5x^3-2x-3=0$ (ix) $\frac{3x}{4}-\frac{5x^2}{8}=\frac{7}{8}$

(x) $\sqrt{x}+\frac{1}{\sqrt{x}}=4$ (xi) $(x+1)(x+3=0$ (xii) $(2x+1)(3x+2)=6(x-1)(x-2)$

(xiii) $16x^2-3=(2x+5)(5x-3)$ (xiv) $(x-2)^2+1=2x-3$ (xv) $x(x+1)+8=(x+2)(x-2)$

(xvi) $x(2x-3)=x^2+1$ (xvii) $(x+2)^3=x^3-4$ (xviii) $x^2+\frac{2}{x^2}=3$

2. Represent each of the following situations in the form of a quadratic equation :

(i) The sum of the squares of two consecutive positive integers is 545. We need to find the integers.

(ii) The hypotenuse of a right triangle is 25 cm. The difference between the length of the other two sides of the triangle is 5 cm. We need to find the lengths of these sides.

(iii) One year ago, the father was 8 times as old as his son. Now his age is square of the son's age. We need to find their present ages.

(iv) Ravi and Raj together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out the number of toys produced on that day.

(v) A cottage industry product a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys product in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

3. In each of the following determine whether the given values are the solutions of the given equation or not :

(a) (i) $x^2 - 7x + 12 = 0$; x = 3, x = 4

(ii) $x^2-\sqrt{2}x-4=0$; $x=-\sqrt{2}, x=-2\sqrt{2}$

(iii) $10x-\frac{1}{x}=3$; $x\neq 0, x=\frac{1}{2}, x=\frac{-1}{2}$

(iv) $\frac{a}{(x-b)}+\frac{b}{(x-a)}=2$; $(x\neq a,b)$; $x=(a+b), x=\frac{a+b}{2}$

(b) (i) $x^2-(\sqrt{2}+\sqrt{3})x+\sqrt{6}=0$; $x=\sqrt{2}, x=\sqrt{3}$

(ii) $\frac{x}{a}+\frac{b}{x}=\frac{a+b}{a}$; $(x\neq 0), x=a, x=b$

4. In each of the following find the value of k for which the given value is a solution of the given equation :

(i) $(x+3)(2x-3k)=0$: $x=6$ (ii) $3\sqrt{7}x^2-4x+k=0$: $x=\frac{\sqrt{7}}{3}$

5. Find the value of p and q for which $x=\frac{2}{3}$ and x = – 3 are the roots of the equation $px^2 + 7x + q = 0$.

SHORT ANSWER TYPE QUESTIONS

Find the solutions of the following quadratic equations by factorization method and check the solutions (1-24) :

1. $27x^2 - 12 = 0$

2. $3\left(\frac{x}{2}+1\right)^2 = 27$

3. $16(x - 4)^2 = 9(x + 3)^2$

4. $x^2 - 300 = 0$

5. $x^2 + (a - b)x = ab$

6. $(3x + a)(3x + b) = ab$

7. $x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$

8. $3\sqrt{7}x^2 + 4x - \sqrt{7} = 0$

9. $\sqrt{3}y^2 + 11y + 6\sqrt{3} = 0$

10. $abx^2 - (a^2 + b^2)x + ab = 0$

11. $x^2 - \frac{x}{12} - \frac{1}{12} = 0$

12. $x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

13. $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

14. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

15. $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

16. $\frac{5}{x-5} + \frac{4}{x} = \frac{3}{x-3}$

17. $\frac{5}{x-5} + \frac{2}{x-2} = \frac{3}{x-3} + \frac{4}{x-4}$

18. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$

19. $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

20. $\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$

21. $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$

22. $\frac{4x-3}{2x+1} - 10\left(\frac{2x+1}{4x-3}\right) = 3$

23. $2\left(\frac{x+2}{2x-3}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$

24. $\frac{x}{2x+1} + \frac{2x+1}{x} = \frac{58}{21}$

Find the roots of each of the following quadratic equations by the method of completing the squares (25 - 29)

25. $x^2 - 6x + 4 = 0$

26. $2x^2 - 5x + 3 = 0$

27. $\sqrt{5}x^2 + 9x + 4\sqrt{5} = 0$

28. $(5z + 2a)(3z + 4b) = 8ab$

29. $2\sqrt{2}x^2 + \sqrt{15}x + \sqrt{2} = 0$

30. Find the solutions of $3x^2 - 2\sqrt{6}x + 2 = 0$ by the method of completing the squares when
(i) x is a rational number (ii) x is a real number

31. Find the solutions of $15x^2 + 3 = 17x$, when (i) x is a rational number (ii) x is a real number.

32. Find the solutions of $5x^2 - 6x - 2 = 17x$, when (i) x is a rational number (ii) x is a real number.

Find the roots of each of the following quadratic equations by using the quadratic formula (33 - 50) :

33. $4x^2 + 3x + 5 = 0$

34. $x^2 - 16x + 64 = 0$

35. $3x^2 - 5x + 2 = 0$

36. $2x^2 2\sqrt{2}x + 1 = 0$

37. $3x^2 2\sqrt{5}x - 5 = 0$

38. $3a^2x^2 + 8abx + 4b^2 = 0,\ a \neq 0$

39. $x + \frac{1}{x} = 3,\ x \neq 0$

40. $\frac{x-2}{4} = \frac{x+2}{x},\ x \neq 0$

41. $y - \frac{15}{4y} + 1 = 0,\ y \neq 0$

42. $\frac{x-3}{x+3} - \frac{x+3}{x-3} = 6\frac{6}{7},\ x \neq -3, 3$

43. $\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{5}{2},\ x \neq 2, -2$

44. $\frac{x}{x-1} + \frac{x-1}{x} = 4,\ x \neq 0, 1$

45. $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

46. $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3,\ x \neq 1, \frac{3}{2}$

47. $(x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = 0$

48. $(y^2 - 4y)^2 + 11(y^2 + 4y) + 28 = 0$

49. $2\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 2 = 0,\ x \neq -1$

50. $(x^2 + 3x + 2)^2 - 8(x^2 + 3x) - 4 = 0$

51. Find the nature of the roots of the following equations. If the real roots exist, find them :

(a) (i) $6x^2 + x - 2 = 0$ (ii) $2x^2 + 5\sqrt{3}x + 6 = 0$ (iii) $2x^2 - 6x + 3 = 0$ (iv) $3a^2x^2 + 8abx + 4b^2 = 0,\ a \neq 0$

(b) Find the discriminate of roots of equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

(c) What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

52. Find the value of k for each of the following quadratic equations, so that they have two real and equal roots :

(a) (i) $2x^2 + kx + 3 = 0$ (ii) $kx^2 - 2\sqrt{5}x + 4 = 0$ (iii) $4x^2 - 2(k + 1)x + (k + 4) = 0$ (iv) $(k - 3)x^2 + 4(k - 3)x + 4 = 0$

(b) (i) $x^2 - 2(k + 1)x + k^2 = 0$ (ii) $(k + 4)x^2 + (x + 1)x + 1 = 0$ (iii) $kx^2 - 2\sqrt{5}x + 4 = 0$ (iv) $2kx^2 - 40x + 25 = 0$

(c) (i) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ (ii) $x^2 - kx + 4 = 0$ (iii) $2x^2 - (k - 2)x + 1 = 0$

53. Determine the value(s) of p for which the quadratic equation $2x^2 + px + 8 = 0$ has (i) real roots.

54. Show that the equation $x^2 + px - 1 = 0$ has (i) real and distinct roots for all real values of p.

55. (a) If – 2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quadratic equation $2x^2 + px + k = 0$ has equal roots, find the value of k.

(b) If – 2 is a root of the quadratic equation $x^2 + px + 2 = 0$ and the quation $2x^2 + px + q = 0$ has equal roots, find the value of p and q.

56. If the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$ or ad = ac.

57. Prove that both the roots of the equation $(x + a)(x + b) + (x + b)(x + c) + (x + c)(x + a) = 0$ are always real and can not be equal unless a = b = c.

58. (a) If the root of the equation $x^2 + 2cx + ab = 0$ are real and unequal, prove that the equation $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ has no real roots.

(b) Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$.

(c) If p,q,r and s are real number such that pr = 2(q + s) than show that atleast one of the equation $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ has real roots.

SBJECTIVE	ANSPER KEY	EXERCISE -2 (x)-CBSE

- **VERY SHORT ANSWER TYPE QUESTIONS**

1. Equations in questions No. (i), (ii), (iv), (vii), (ix), (xiii), (xiv), (xvi) and (xvii) are quadratic equations.

2. (i) $x^2 + x - 272 = 0$, where x is the smaller integer. (ii) $x^2 + 5x - 300 = 0$, where x is the length of one side.
(iii) $x^2 - 8x + 7 = 0$, where x (in years) is the present age of son.
(iv) $x^2 - 45x + 324 = 0$, where x is the number of marbles with Ravi.
(v) $x^2 - 55x + 750 = 0$, where x (in km/h) is the speed of the train.

3. (a) (i) Both are solution (ii) $x = -\sqrt{2}$ is a solution but $x = -2\sqrt{2}$ is not a solution.
(iii) $x = \frac{1}{2}$ is a solution but $x = \frac{-1}{2}$ is not a solution. (iv) Both are solution
(b) (i) Both are solution (ii) Both are solution

4. (i) k = 4, (ii) $k = -\sqrt{7}$ **5.** p = 3, q = – 6

- **SHORT ANSWER TYPE QUESTIONS**

1. $\frac{2}{3}, -\frac{2}{3}$ **2.** 4, -8 **3.** 1, 25 **4.** $10\sqrt{3}, -10\sqrt{3}$ **5.** b, - a **6.** $0, -\frac{(a+b)}{3}$ **7.** $\sqrt{2}$ **8.** $-\frac{\sqrt{7}}{3}, \frac{\sqrt{7}}{7}$ **9.** $-\frac{2}{\sqrt{3}}, -3\sqrt{3}$
10. $\frac{a}{b}, \frac{b}{a}$ **11.** $\frac{1}{3}, -\frac{1}{4}$ **12.** $a, \frac{1}{a}$ **13.** $\frac{-a}{a+b}, -\frac{(a+b)}{a}$ **14.** –a, –b **15.** – 1 **16.** 12, – 2 **17.** $0, \frac{7}{2}$ **18.** $-\frac{5}{2}, \frac{3}{2}$
19. 5, – 1 **20.** $6, \frac{40}{13}$ **21.** $-10, -\frac{1}{5}$ **22.** $-\frac{4}{3}, \frac{1}{8}$ **23.** $\frac{11}{5}, \frac{5}{8}$ **24.** $3, -\frac{7}{11}$ **25.** $3 \pm \sqrt{5}$ **26.** $1, \frac{3}{2}$ **27.** $-\sqrt{5}, \frac{-4}{\sqrt{5}}$
28. $0, -\frac{6a+20b}{15}$ **29.** No solution **30.** (i) No solution (ii) $\sqrt{\frac{2}{3}}$ **31.** (i) No solution (ii) $\frac{17+\sqrt{109}}{30}, \frac{17-\sqrt{109}}{30}$
32. (i) No solution (ii) $\frac{3 \pm \sqrt{19}}{5}$ **33.** No solution **34.** 8 **35.** $1, \frac{2}{3}$ **36.** $\frac{1}{\sqrt{2}}$ **37.** $\frac{\sqrt{5}}{3}, -\sqrt{5}$ **38.** $\frac{-2b}{a}, \frac{-2b}{3a}$
39. $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$ **40.** $3+\sqrt{17}, 3-\sqrt{17}$ **41.** $\frac{3}{2}, -\frac{5}{2}$ **42.** $-4\frac{9}{4}$ **43.** 6, – 6 **44.** $\frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$ **45.** $\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$
46. $\frac{1}{2}, \frac{4}{3}$ **47.** –1, 3, $1+\sqrt{2}, 1-\sqrt{2}$ **48.** – 2, – 1, $\frac{-3 \pm \sqrt{21}}{2}, \frac{-3 \pm \sqrt{5}}{2}$ **49.** – 2, 1 **50.** 1, 0, – 3 – 4
51. (a) (i) $\frac{1}{2}, \frac{-2}{3}$ (ii) $\frac{-\sqrt{3}}{2}, -2\sqrt{3}$ (iii) $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$ (iv) $\frac{-2b}{a}, \frac{-2b}{3a}$ (b) $\frac{1}{3}, \frac{1}{3}$ (c) Roots are real and unequal
52. (a) (i) $k = \pm 2\sqrt{6}$ (ii) $k = \frac{5}{4}$ (iii) = – 3, 5 (iv) k = 4 ; (b) (i) $k = \frac{-1}{2}$ (ii) k = 5, – 3 (iii) $k = \frac{5}{4}$ (iv) k = 8
(c) (i) k = 14 (ii) k = ± 4 (iii) $k = 2 \pm \sqrt{2}$ **53.** (i) p = ± 8 (ii) p ≤ -8 or p ≥ 8, p $\in$ R **55.** $k = \frac{9}{4}$,(b) p = 3, $q = \frac{9}{8}$

EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

APPLICATIONS TO WORD PROBLEMS

1. Find the numbers whose sum is 40 and product 375.

2. The difference between two integers is 4. Their product is 221. Find the numbers.

3. The sum of a natural number and its reciprocal is $\frac{65}{8}$. Find the natural numbers.

4. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$.

5. The sum of two numbers is 12 and the sum of their squares is 74. Find the natural numbers.

6. Find two consecutive natural numbers, the sum of whose squares is 145.

7. Find two consecutive positive even integers, whose product is 224.

8. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

9. Find two consecutive multiples of 3 whose product is 270.

10. A number consists of two digits whose product is 18. If 27 is added to the number, the digits interchange their places. Find the number

11. A two-digit number contains the smaller of the two digits in the unit place. The product of the digits is 40 and the difference between the digits is 3. Find the number.

12. The sum of numerator and denominator of a certain fraction is 10. If 1 is subtracted from both the numerator and denominator, the fraction is decreased by $\frac{2}{21}$. Find the fraction.

13. Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.

14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time taken by the first pipe alone to fill the tank.

15. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump?

16. Albert goes to his friend's house which is 15 km away from his house. He covers half of the distance at a speed of x km/hr and the remaining at (x + 2) km/hr. If he takes 2 hrs 30 min. to cover the whole distance, find x.

17. (i) A train covers a distance of 780 km at x km/hr. Had the speed been (x – 5) km/hr, the time taken to cover the same distance would have been increased by 1 hour. Write down an equation in x and solve it to evaluate x.

(ii) A train covers a distance of 600 km at x km/hr. Had the speed been (x + 20) km/hr, the time taken to cover the same distance would have been reduced by 5 hour. Write down an equation in x and solve it to evaluate x.

18. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car.

19. The distance by road between two towns A and B, is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car.

(i) Write down the time taken by the car to reach town B from A, in terms of x.

(ii) Write down the time taken by the train to reach town B from A, in terms of x.

(iii) If the train takes 2 hours less than the car to reach town B, obtain an equation in x and solve it .

(iv) Hence, find the speed of the train .

20. Car A travels x km for every litre of petrol, while car B travels (x + 5) km for every litre of petrol.

(i) Write down the number of litres used by car A and B in covering a distance of 400 km.

(ii) If car A used 4 litres of petrol more than car B in covering 400 km, write an equation in x and solve it to determine the number of litres of petrol used by car B for the journey.

21. The speed of a boat in still water is x km/hr and the speed of the stream is 3 km/hr.

(i) Write the speed of the boat upstream, in terms of x.

(ii) Write the speed of the boat downstream, in terms of x.

(iii) If the boat goes 15 km upstream and 22 km downstream in 5 hours, write an equation in x to represent the statement.

(iv) Solve the equation to evaluate x.

22. The hypotenuse of right triangle is 20 m. If the difference between the lengths of other sides be 4 m. find the other sides.

23. The length of the sides of a right triangle are (2x – 1) m, and (4x + 1) m, where x > 0. Find :

(i) The value of x. (ii) The area of the triangle.

24. Two squares have sides x cm and (x + 5) cm. The sum of their areas is 697 sq. cm.

(i) Express this as an algebraic equation in x.

(ii) Solve this equation to find the sides of the squares .

25. The length of a rectangle is 8 metres more than its breadth and its area is 425 m^2.

(i) Taking x metres as the breadth of the rectangle, write an equation in x that represents the above statement.

(ii) Solve the above equation and find the dimensions of the rectangle.

26. The ratio between the length and the breadth of a rectangular field is 3 : 2. If only the length is increased by 5 metres, the new area of the field will be 2600 sq. metres. What is the breadth of the rectangular field?

27. The perimeter of a rectangular plot of land is 114 metres and its area is 810 square metres.

(i) Take the length of plot as x metres. Use the perimeter 114 m to write the value of the breadth in terms of x.

(ii) Use the values of length, breadth and area to write an equation in x.

(iii) Solve the equation to find the length and breadth of the plot.

28. Write a rectangular garden 10 m wide and 20 m long, we wish to pave a walk around the borders of uniform width so as to leave an area of 96 m^2 for flowers. Haw wide should the walk be ?

29. The area of right-angle triangle is 96 m^2. If the base is three times its altitude, find the base.

30. The length of the parallel sides of trapezium are (x + 8) cm and (2x +3) cm, and the distance between them is (x + 4) cm. If its area is 590 cm^2, find the value of x.

31. A man buys an article for Rs. x and sells it for Rs. 56 at a gain of x%. Find the value of x.

32. Rohit is on tour and he has Rs. 360 for his expenses. If he exceeds his tour by 4 days, he must cut down his daily expenses by Rs. 3. For how many days Rohit is on tour?

33. Rs. 6400 were divided equally among x persons. Had this money been divided equally among (x + 14) persons, each would have got Rs. 28 less. Find the value of x.

34. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased Rs. 10. Find how many students went for the picnic.

35. A shopkeeper buys x books for Rs. 720.

(i) Write the cost of 1 book in terms of x.

(ii) If the cost of per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more. Write down the equation in x for the above situation and solve it to find x.

36. A piece of cloth costs Rs. 35. If the length of the piece would have been 4 m longer and each metre costs Rs. 1 less, the cost would have remained unchanged. How long is the piece?

37. A fruit seller-bought x apples for Rs. 1200.

(i) Write the cost price of each apple in terms of x.

(ii) If 10 of the apple were rotten and he sold each of the rest at Rs. 3 more than the cost price of each, write the selling price of (x – 10) apples.

(iii) If he made a profit of Rs. 60 in this transaction, from an equation in x and solve it to evaluate x.

38. Vibha and Sanya distribute Rs. 100 each in charity. Vibha distributes money to 5 more people than Sanya and Sanya gives each Re 1 more than Vibha. How many people are recipients of the charity?

SBJECTIVE | **ANSPER KEY** | **EXERCISE -3 (x)-CBSE**

Applications To Word Problems

1. 15, 25 **2.** 13, 17 or 13, – 17 **3.** 8 **4.** 15, 12 **5.** 5, 7 **6.** 8, 9 **7.** 14, 16 **8.** 27, 29, 31 **9.** 15, 18

10. 36 **11.** 85 **12.** $\frac{3}{7}$ **13.** 29 years, 5 years **14.** 15 hours **15.** 50 m^3/ min **16.** x = 4

17. (i) $x^2 - 5x - 3900 = 0$, x = 65 (ii) $x^2 + 20x - 2400 = 0$, x = 40 **18.** 30 km/hr

19. (i) $\frac{216}{x}$ hrs (ii) $\frac{208}{(x+16)}$ hrs (iii) $x^2 + 12x - 1728 = 0$, x = 36 (iv) 52 km/hr

20. (i) $\left(\frac{400}{x}\right)$ litres and $\left(\frac{400}{x+5}\right)$ litres (ii) $\frac{400}{x+5} - \frac{400}{(x+5)} = 4, x = 20$. Car B consumed 16 litres.

21. (i) (x – 3) km/hr (ii) x + 3 km/hr (iii) $\frac{15}{(x+3)} + \frac{22}{(x+3)} = 5$ (iv) x = 8 **22.** 16 m, 12,

23. (i) x = 3 (ii) 30 m^2 **24.** (i) $x^2 + 5x - 336 = 0$ (ii) 16 cm, 21 cm 25. (i) $x^2 + 8x - 425 = 0$ (ii) 17 m, 25 m

26. 40 m **27.** (i) Breadth = (57 – x) m (ii) $x^2 - 57x + 810 = 0$ (iii) ℓ = 30 m, b = 27 m **28.** 2 m **29.** 24 m

30. x = 16 **31.** x = 40 **32.** 20 days **33.** x = 50 **34.** 16 **35.** (i) Rs. $\left(\frac{720}{x}\right)$ (ii) $x^2 + 2x - 228 = 0$ x = 16 **36.** 10 m

37. (i) Rs. $\left(\frac{1200}{x}\right)$ (ii) Rs. (x – 10) $\left(\frac{1200}{x} + 3\right)$ (iii) $x^2 - 30x - 4000 = 0$, x = 80 **38.** 45

EXERCISE – 4 (FOR SCHOOL/BOARD EXAMS)

PREVIOUS YEARS BOARD QUESTIONS

SHORT ANSWER TYPE QUESTIONS

1. Find the values of k so that (x – 1) is a factor of $k^2x^2 - 2kx\ 3$. **[CBSE-Delhi-2003]**
2. Solve using the quadratic formula : $x^2 - 4x + 1 = 0$ **[ICSE-2003]**
3. Solve for x : $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$ **[CBSE-Delhi-2004]**
4. Solve for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE-Delhi-2004]**
5. Solve for x : $9x^2 - 9(a + b)x = [2a^2 + 5ab + 2b^2] = 0$ **[CBSE-Delhi-2004]**
6. Using quadratic formula, solve the following quadratic equation for x : $p^2x^2 + (p^2 - q^2)x - q^2 = 0$ **[CBSE-Al-2004]**
7. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 2x + (a^2 - b^2) = 0$ **[CBSE-Al-2004]**
8. Using quadratic formula, solve the following quadratic equation for x : $x^2 - 4x + 4a^2 - b^2 = 0$ **[CBSE-Al-2004]**
9. Solve for x : $9x^2 - 6a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**
10. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$ **[CBSE-Foreign-2004]**
11. Solve for x : $16x^2 - 8a^2x + (a^4 - b^4) = 0$ **[CBSE-Foreign-2004]**
12. Solve for x : $36x^2 - 12ax + (a^2 - b^2) = 0$ **[CBSE-Delhi-2004C]**
13. Solve the equation $3x^2 - x - 7 = 0$ and give your answer correct to two decimal places. **[ICSE-2004]**
14. Solve for x : $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$ **[CBSE-Foreign-2005]**

    OR

    Solve for x : $x^2 - 2(a^2 + b^2)x + (a^2 - b^2)^2 = 0$ **[CBSE-Delhi-2006C]**
15. Solve $x^2 - 5x - 10 = 0$ and give your answer correct to two decimal places **[ICSE-2005]**
16. Using quadratic formula, solve for x : $9x^2 - 3(a + b)x + ab = 0$

    OR

    The sum of the square of two consecutive natural numbers is 421. Find the numbers. **[CBSE-Dehli-2005C]**
17. Using quadratic formula, solve the following for x : $9x^2 - 3(a^2 + b^2)x + a^2b^2 = 0$

    OR

    The sum of the square of three consecutive positive integers is 50. Find the integers. **[CBSE-Al-2005C]**
18. Rewrite the following as a quadratic equation in x and then solve for x : $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$ **[CBSE-Al-2006C]**
19. Solve $2x - \frac{1}{x} = 7$ and give your answer correct to 2 decimal places. **[ICSE-2006]**
20. Solve $x^2 - 3x - 9 = 0$ and give your answer correct to 2 decimal places. **[ICSE-2007]**
21. Find the roots of the following equation : $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4, 7$ **[CSBE-Delhi-2008]**
22. Is x = – 2 a solution of the equation $x^2 - 2x + 8 = 0$? **[CSBE-Al-2008]**
23. Is x = – 3 a solution of the equation $2x^2 + 5x + 3 = 0$? **[CSBE-Al-2008]**
24. Is x = – 4 a solution of the equation $2x^2 + 5x - 12 = 0$? **[CSBE-Al-2008]**

25. Show that x = – 3 is a solution of $x^2 + 6x + 9 = 0$. **[CSBE-Foreign-2008]**
26. Show that x = – 3 is a solution of $2x^2 + 6x – 3 = 0$. **[CSBE-Foreign-2008]**
27. Show that x = – 2 is a solution of $3x^2 + 13x + 14 = 0$. **[CSBE-Foreign-2008]**
28. Find the discriminant of the equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$. **[CSBE-Al-2009]**
29. The sum of two numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$. **[CSBE-Al-2009]**
30. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$. **[CSBE-Foreign-2009]**

LONG ANSWER TYPE QUESTIONS

1. An aeroplane traveled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for (i) the onward journey, (ii) the return journey. If the return journey took 30 minutes less than the onward journey, write an equation in x and find the value of x. **[ICSE-2002]**
2. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find (i) the number of rows in the original arrangement, (ii) the number of seats in the auditorium after rearrangement. **[ICSE-2003]**
3. Solve for x : $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$; given that $x \neq -3,\ x \neq \frac{1}{2}$ **[CSBE-Delhi-2004]**
4. Solve for x : $2\left(\frac{x-1}{x+3}\right) - 7\left(\frac{x+3}{x-1}\right) = 5$; given that $x \neq -3,\ x \neq 1$ **[CSBE-Delhi-2004]**
5. Solve for x : $2\left(\frac{2x+3}{2x+1}\right) - 10\left(\frac{2x+1}{2x-3}\right) = 3$; given that $x \neq 3,\ x \neq \frac{-3}{2}$ **[CSBE-Delhi-2004]**
6. Solve for x : $2\left(\frac{4x-3}{2x+1}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$; given that $x \neq \frac{-1}{2};\ x \neq \frac{3}{4}$

   OR

   300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students. **[CSBE-Al-2004]**
7. Solve for x : $2\left(\frac{x+2}{2x-3}\right) - 9\left(\frac{2x-3}{x+2}\right) = 3$; given that $x \neq \frac{3}{2};\ x \neq -2$

   OR

   An aeroplane takes one hour less for a journey of 1200 km if its speed is increased by 100 km/hour from its usual speed. Find the its usual speed. **[CSBE-Foreign-2004]**
8. A two digit number is four times the sum of its digits and is also equal to twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**
9. A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number **[CSBE-Delhi-2004C]**
10. A two digit number is 5times the sum of its digits and is also equal to 5 more than twice the product of its digits. Find the number **[CSBE-Delhi-2004C]**

11. The sum of two number a and b is 15, and the sum of their reciprocals $\frac{1}{a}$ and $\frac{1}{b}$ is $\frac{3}{10}$. Find the number **[CSBE-Delhi-2005]**

12. The sum of two number is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the number **[CSBE-Delhi-2005]**

13. The sum of two number is 18. The sum of their reciprocals is $\frac{1}{4}$. Find the number **[CSBE-Delhi-2005]**

14. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

15. A two digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

16. A two digit number is such that the product of its digits is 14. If 45 is added to the number, the digits interchange their places. Find the number **[CSBE-Al-2005]**

17. The sum of the square of two natural number is 34. If the first number is one less than twice the second number, find the number **[CBSE-Foreign-2005]**

18. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hour from its usual speed. Find the usual speed of the train. **[CSBE-Delhi-2005C, 2006]**

19. Solve for $x\frac{x+1}{x-1}+\frac{x-2}{x+2}=3:(x\neq 1,-2)$ **[CSBE-Al-2005C]**

OR

Aeroplane left 30 minutes later than its scheduled time and in order to reach destination 1500 km away in time, it has to increase its speed by 250 km/h from its usual speed. Determine its usual speed.

20. Solve for x : $\frac{1}{a+b+x}+\frac{1}{a}+\frac{1}{x}:a\neq 0,b\neq 0,x\neq 0$

OR

Solve for x : $abx^2 + (b^2 - ac)x - bc = 0$ **[CSBE-Delhi-2005]**

21. Solve for x : $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

OR

Solve for x : $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}\ (x\neq 2,4)$ **[CSBE-Al-2005]**

22. By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. **[ICSE-2005]**

23. Solve for x : $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

OR

A two digit number is such that the product of its digits is 35. When 18 is added to number, the digits interchange their places. Find the number. **[CBSE-Dehli-2006]**

24. Using quadratic formula, solve the equation : $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

OR

The sum of two natural numbers is 8. Determine the numbers if the sum of their reciprocals is $\frac{8}{15}$.

[CBSE-Al-2006]

25. Solve for x : $(a + b)^2 x^2 + 8 (a^2 – b^2) x + 16 (a – b)^2 = 0$

OR

Two number differ by 3 and their product is 504. Find the number. **[CBSE-Foreign-2006]**

26. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speeds of the two trains. **[CBSE-Foreign-2006]**

27. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages. **[CBSE-Delhi-2006C]**

28. A 2-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

OR

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train **[CBSE-Al-2006C]**

29. A shopkeeper buys x books for Rs. 720. (i) Write the cost of 1 book in terms of x, (ii) If the cost per book were Rs. 5 less, the number of books that could be bought for Rs. 720 would be 2 more.
Write down the equation in x for the above situation and solve it to find x. **[ICSE-2006]**

30. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1200. Find the original list price of the book. **[CBSE-Delhi-2007]**

31. The numerator of a fraction is one less than its denominator. If three is added to each of the numerator and denominator, the fraction is increased by $\frac{3}{28}$. Find the fraction.

OR

The difference of squares of two natural numbers is 45. The square of the smaller umber is four times the larger number. Find the numbers. **[CBSE-Al-2007]**

32. Some students planned a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member increased by Rs. 10. Find how many students went for the picnic. **[ICSE-2008]**

33. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

OR

The sum of the areas of two squares is 640 m^2. If the difference in their perimeters be 64 m, find the sides of the two squares. **[CBSE-Delhi-2008]**

34. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. **[CBSE-Al-2008]**

35. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to it's hole at the base of pillar. Seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?

OR

Two difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers.

[CBSE-Foreing-2008]

36. The sum of the squares of two consecutive odd numbers is 394. Find the numbers. **[CBSE-Delhi-2009]**

37. Solve the following equation for x : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

OR

If (– 5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k. **[CBSE-Al-2009]**

38. A trader bought a number of articles for Rs. 900. Five articles were found damaged. He sold each of the remaining articles at Rs. 2 more than what he paid for it. He got a profit of Rs. 80 on the whole transaction. Find the number of articles he bought.

OR

Two years ago a man's age was three times the square of his son's age. Three years hence his age will be four times his son's age. Find their present ages. **[CBSE-Foreing-2009]**

39. A girl is twice as old as her sister. Four years hence. The product of their ages (in yeras) will be 160. Find their present ages. **[CBSE-Al-2010]**

SBJECTIVE	ANSPER KEY	EXERCISE -4 (x)-CBSE

- **SHORT ANSWER TYPE QUESTION**

1. (–1, 3) **2.** $[2+\sqrt{3}, 2-\sqrt{3}]$ **3.** $x=\frac{a^2}{2},\frac{b^2}{2}$ **4.** $x=\frac{(a^2+b^2)}{2}.\frac{(a^2-b^2)}{2}$ **5.** $x=\frac{(2a+b)}{3}.\frac{(a+2b)}{3}$ **6.** $\frac{q^2}{p^2},1$

7. a + b, a – b **8.** 2a + b, 2a – b **9.** $\frac{(a^2+b^2)}{3},\frac{(a^2-b^2)}{3}$ **10.** $\frac{(a+b)}{3}\frac{(a-b)}{3}$ **11.** $\frac{(a^2+b^2)}{4},\frac{(a^2-b^2)}{4}$

12. $\frac{(a+b)}{6}\frac{(a-b)}{6}$ **13.** 1.70, – 1.37 **14.** $\frac{\sqrt{3}}{4},\frac{-2}{\sqrt{3}}$ or $(a+b)^2,(a-b)^2$ **15.** 6.53, – 1.53 **16.** $\frac{a}{3}.\frac{b}{3}$ or 14, 15

17. $\frac{a^2}{3}.\frac{b^2}{3}$ or 3, 4, 5 **18.** x = – 2, 1 **19.** 3.64 – 0.14 **20.** 4.85, – 1.85 **21.** 2, 1 **22.** No **23.** No

24. Yes 28.64 29. 3and 5

- **LONG ANSWER TYPE QUESTION**

1. (i) $\left(\frac{400}{x}\right)$hrs. (ii) $\left(\frac{400}{x+40}\right)$ hrs; x = 160 km/hr 2. (i) 30 (ii) 1200 3. x = – 10 1/5 4. $x=-\frac{23}{5},-1$

6. $x=-\frac{4}{3},\frac{1}{8}$ or 50 7. $x=\frac{5}{8},\frac{11}{5}$ or 300 km/hr 8. 36 9. 84 10. 45 11. 5,10 12. 4,12 13. 6,12 14. 35 15. 45

16. 27 17. 5 and 3 18. 25 km/hr 19. x = – 5, 2 or 750 km/hr 20. x = – a, – b or $x=\frac{c}{b},\frac{-b}{a}$

21. $x=\frac{1}{b^2},-\frac{1}{a^2}$ or $x=\frac{5}{2},5$ 22. 30 km/hr 23. $x=\frac{-2b}{3a},\frac{3a}{4b}$ or 57 24. $x=\frac{-3a^2}{b^2},\frac{4b^2}{a^2}$ or 3 and 5

25. $x=\frac{-4(a-b)}{a+b}$ or 21, 24 or – 21, – 24 26. 40 km/hr, 50 km/hr 27. 9 years, 27 years 28. 92 or 45 km/hr

29. (i) Rs. $\left(\frac{720}{x}\right)$ (ii) x2 + 2x – 288 = 0, x = 16 **30.** 10 and 5 or Rs. 30 **31.** 3/4 or 9 and 6 **32.** 16

33. Marks in maths : 12(9), Marks in science : 16(19) **34.** 6 km/hr or 25 hrs and 15 hrs

35. 12 m or (7 and 3) or (– 3 and – 7) **36.** 13 and 15 **37.** $\frac{2a+b}{3},\frac{a+2b}{3}$ or p = 7 and $k=\frac{7}{4}$

38. 75 or son's age = 5 years and man's age = 29 years. **29.** 6 years and 12 years

EXERCISE – 5 (FOR OLYMPIADS)

CHOOSE THE CORRECT ONE

1. The roots of the equation (x – a) (x – b) (x – c) + (x – b) (x – c) +(x – c) (x – a) = 0 are :
(A) Real (B) Not real (C) Imaginary (D) Rational

2. The integral values of k for which the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both the roots real, distinct and negative is :
(A) 0 (B) 2 (C) 3 (D) – 4

3. If the roots of the equation $\frac{x^2 - bx}{ac - c} = \frac{m-1}{m+1}$ are equal and of oppositve sign, then the value of m will be :
(A) $\frac{a-b}{a+b}$ (B) $\frac{b-a}{a+b}$ (C) $\frac{a+b}{a-b}$ (D) $\frac{b+a}{b-a}$

4. If α, β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3}\frac{1}{\beta^3}$ is equal to :
(A) $-\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 32 (D) $\frac{1}{32}$

5. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + ...to\infty}}}$, then :
(A) x is an irrational number (B) 2 < x < 3 (C) x = 3 (D) None of these

6. If α, β are the roots of the equation $x^2 + 7x + 12 = 0$, then the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is :
(A) $x^2 + 50x + 49 = 0$ (B) $x^2 - 50x + 49 = 0$ (C) $x^2 - 50x - 49 = 0$ (D) $x^2 + 12x + 7 = 0$

7. The values of k (k > 0) for which the equation $x^2 + kx + 64 =$ and $x^2 - 8x + k = 0$ both will have real roots is :
(A) 8 (B) 16 (C) – 64 (D) None of these

8. If α, β are the roots of the equation $x^2 + bx - c = 0$, then the equation whose roots are b and c is :
(A) $x^2 + \alpha x - \beta = 0$ (B) $x^2 - [(\alpha+\beta) + \alpha\beta]x - \alpha(\alpha+\beta) = 0$
(C) $x^2 + (\alpha\beta + \alpha + \beta)x + \alpha\beta(\alpha+\beta) = 0$ (D) $x^2 + (\alpha\beta + \alpha + \beta)x - \alpha\beta(\alpha+\beta) = 0$

9. Solve for y : $9y^4 - 29y^2 + 20 = 0$
(A) $\pm 2, \pm\frac{2}{3}$ (B) $\pm 3, \pm\frac{3}{\sqrt{5}}$ (C) $\pm 1, \pm\frac{2\sqrt{5}}{3}$ (D) None of these

10. Solve for x : $x^6 - 26x^3 - 27 = 0$
(A) – 1, 3 (B) 1, 3 (C) 1, – 3 (D) – 1, – 3

11. Solve : $\sqrt{2x+9} + x = 3$:
(A) 4, 16 (B) 8, 20 (C) 2, 8 (D) None of these

12. Solve : $\sqrt{2x+9} - \sqrt{x-4} = 3$
(A) 4, 16 (B) z8, 20 (C) 2, 8 (D) None of these

13. Solve for x : $2\left[x^2 + \frac{1}{x^2}\right] - 9\left[x + \frac{1}{x}\right] + 14 = 0$:
(A) $\frac{1}{2}$, 1, 2 (B) 2, 4, $\frac{1}{3}$ (C) $\frac{1}{3}$, 4, 1 (D) None of these

14. Solve x : $6\left[x^2+\frac{1}{x^2}\right]-25\left(x+\frac{1}{x}\right)+12=0$:

(A) $-\frac{1}{3},-\frac{1}{2}$, 2, 3 (B) $\frac{1}{3},\frac{1}{2}$, 2, 3 (C) $\frac{1}{3},\frac{1}{2}$, – 2 , – 3 (D) None of these

15. Solve for x : $\sqrt{x^2+x-6}-x+2=\sqrt{x^2-7x+10},\ x\in R$:

(A) 2, 6, $-\frac{10}{3}$ (B) 2, 6 (C) – 2 , – 6 (D) None of these

16. Solve for x : $3^{x+2}+3^{-x}=10$

(A) – 3, – 2 (B) – 2, 0 (C)2 , 3 (D) None of these

17. Solve for x : (x + 1) (x + 2) (x + 3) (x + 4) = 24 (x $\in$ R) :

(A) 0, – 5 (B) 0, 5 (C) 0, – 2 (D) 0, 2

18. The sum of all the real roots of the equation $|x-2|^2+|x-2|^2-2=0$ is :

(A) 2 (B) 3 (C) 4 (D) None of these

19. If a, b $\in$ {1, 2, 3, 4}, then the number of quadratic equation of the form ax2 + bx + 1 = 0, having real roots is :

(A) 6 (B) 7 (C) 8 (D) None of these

20. The number of real solutions of $x-\frac{1}{x^2-4}=2-\frac{1}{x^2-4}$ is :

(A) 0 (B) 1 (C) 2 (D) Infinite

21. If $(2+\sqrt{3})^{x^2-2x+1}+(2-\sqrt{3})^{x^2-2x-1}=\frac{2}{2-\sqrt{3}}$, then x is equal to :

(A) 0 (B) 1 (C) 2 (D) Both (A) and (C)

22. The quadratic equation $3x^2 + 2(a^2 + 1) x + a^2 - 3a + 2 = 0$ possesses roots of opposite sign then a lies in :

(A) $(-\infty,0)$ (B) $(-\infty,1)$ (C) (1, 2) (D) (4, 9)

23. The equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ has :

(A) No solution (B) One solution (C) Two solution (D) More than two solution

24. The number of real solutions of the equation $2|x|^2-5|x|+2=0$ is :

(A) 0 (B) 4 (C) 2 (D) None of these

25. The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$:

(A) 0 (B) 2 (C) 3 (D) 6

26. The number of real solutions of the equation $2^{3x^2-7x+4}=1$ is :

(A) 0 (B) 4 (C) 2 (D) Infinitely many

27. If the equation $(3x)^2 + (27\times 3^{1/k} - 15)\,x + 4 = 0$ has equal roots, then k =

(A) – 2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 0

28. If $x=\sqrt{2+\sqrt{2+\sqrt{2+...+\infty}}}$, then x is :

(A) 1 (B) 2 (C) 3 (D) None of these

29. Equation $ax^2 + 2x + 1$ has one double root if :

(A) a = 0 (B) a = – 1 (C) a = 1 (D) a = 2

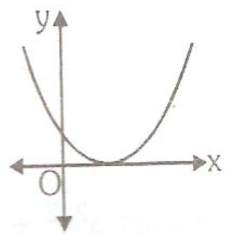
30. Solve for x : (x + 2) (x – 5) (x – 6) (x + 1) = 144 :

(A) –1, –2, –3 (B) 7, – 3, 2 (C) 2, – 3, 5 (D) None of these

31. If $f(x) = \frac{2x+5}{x^2+x+5}$, then find f(f(– 1))

(A) $\frac{149}{155}$ (B) $\frac{155}{147}$ (C) $\frac{155}{149}$ (D) $\frac{147}{155}$

32. What does the following graph represent?



(A) Quadratic polynomial has just one root.

(B) Quadratic polynomial has equal one roots.

(C) Quadratic polynomial has no root.

(D) Quadratic polynomial has equal roots and constant term is non-zero.

33. Consider a polynomial $ax^2 + bx + c$ such that zero is one of it's roots then :

(A) $c = 0, x = \frac{-b}{a}$ satisfies the polynomial equation

(B) $c \neq 0, x = \frac{-b}{a}$ satisfies the polynomial equation

(C) $, x = \frac{-b}{a}$ satisfies the polynomial equation

(D) Polynomial has equal roots.

34. For a parabola opening upwards and above x-axis, quadratic will have :

(A) Equal roots and a = 0 (B) Unequal roots and $a \neq 0$

(C) No roots, a > 0 (D) No roots, a < 0

35. The equation $\sqrt{x+10} - \frac{6}{\sqrt{x+10}} = 5$ has :

(A) An extraneous root between – 5 and – 1. (B) An extraneous root between – 10 and – 6.

(C) Two extraneous roots. (D) A real root between 20 and 25.

36. Consider a quadratic polynomial $f(x) = ax^2 - x + c$ such that ac > 1 and it's graph lies below x-axis then :

(A) a < 0, c > 0 (B) a < 0, c < 0 (C) a > 0, c > 0 (D) a > 0, c < 0

37. If α, β are the roots of a quadratic equation $x^2 - 3x + 5 = 0$, then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is :

(A) $x^2 + 4x + 1 = 0$ (B) $x^2 - 4x + 4 = 0$ (C) $x^2 - 4x - 1 = 0$ (D) $x^2 + 2x + 3 = 0$

OBJECTIVE — ANSPER KEY — EXERCISE -5

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	A	B	C	B	B	C	C	A	B	B	A	A	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	A	C	B	A	D	C	A	B	A	C	B	B	C	B
Que.	31	32	33	34	35	36	37								
Ans.	C	D	A	C	B	B	B								

EXERCISE – 6 (FOR IIT-JEE/AIEEE)

CHOOSE THE CORRECT ONE

Based on Graph of Quadratic Expression

1. If the expression $\left[mx - 1 + \frac{1}{x}\right]$ is non negative for all positive real x, then the minimum value of m must be :

(A) $\frac{-1}{2}$ (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

2. The expression $a^2x^2 + bx + 1$ will be positive for all $x \in R$ if :

(A) $b^2 > 4a^2$ (B) $b^2 < 4a^2$ (C) $4b^2 > a^2$ (D) $4b^2 < 4a^2$

3. If x be real, then 3x2 + 14x + 11 > 0 when :

(A) $x < -\frac{3}{2}$ (B) $x > -\frac{3}{4}$ (C) x > – 2 (D) Never

4. For what value of a the curve y = x2 + ax + 25 touches the x-axis :

(A) 0 (B) ± 5 (C) ± 10 (D) None of these

5. The integer k for which the inequality x^2 – 2(4k – 1) x + 15k2 – 2k – 7 > 0 is valid for any x is :

(A) 2 (B) 3 (C) 4 (D) 6

6. The value fo the expression $x^2 - 2bx + c$ will be positive for all real x if :

(A) $b^2 - 4c > 0$ (B) $b^2 - 4c < 0$ (C) $c^2 < b$ (D) $b^2 < c$

7. If the roots fo the quadratic equation $ax^2 + bx + c = 0$ are imaginary then for all values of a, b, c and x R the expression $a^2x^2 + abx + ac$ is :

(A) Positive (B) Non-negative (C) Negative (D) May be positive, zero or negative

Based on Maximum & Minimum Value of the Expression :

8. The range of $y = \frac{x+2}{2x^2+3x+6}$, if x is real, is :

(A) $-\frac{1}{13} \le y \le \frac{1}{3}$ (B) $\frac{1}{13} \le y \le \frac{1}{3}$ (C) $-\frac{1}{13} \le y \le \frac{1}{13}$ (D) None of these

9. If $x \in R$ and $k = \frac{(x^2 - x + 1)}{(x^2 + x + 1)}$, then :

(A) $x \le 0$ (B) $\frac{1}{3} \le k \le 3$ (C) $k \ge 5$ (D) None of these

10. For all real values of x, the maximum value of the expression $\frac{x}{x^2 - 5x + 9}$ is :

(A) 1 (B) 45 (C) 90 (D) None of these

11. If x be real then the maximum and minimum value of the expression $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ are

(A) 2, 1 (B) $7, \frac{1}{7}$ (C) $5, \frac{1}{5}$ (D) None of these

12. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is : **[AIEEE-2006]**

(A) $\frac{17}{7}$ (B) $\frac{1}{4}$ (C) 41 (D) None of these

Based on the Concept of Common Roots :

13. The value of k, so that the equation $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have one root in common is :
(A) – 2, – 3 (B) $-3, -\frac{27}{7}$ (C) – 5, – 6 (D) None of these

14. If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$, then the common factor is :
(A) (x – 3) (B) (x – 6) (C) (x – 8) (D) None of these

15. The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is :
(A) 0, 2 (B) 0, – 2 (C) 2, – 2 (D) None of these

16. If the equation $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$, $(b \neq c)$ have a common root then :
(A) b + c = 0 (B) b + c = 1 (C) b + c + 1 = 0 (D) None of these

17. If both the roots of the equation $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and
$6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then 2r – p is equal to :
(A) 1 (B) – 1 (C) 2 (D) 0

18. If every pair from among the equation $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$ and $x^2 rx + pq = 0$ has a common root, then the sum of three common roots is :
(A) 2 (p + q + r) (B) p + q + r (C) – (p + q + r) (D) pqr

19. If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$; a > 0 have a common root, then a is equal to :
(A) 1 (B) 2 (C) 4 (D) 5

20. The values of a for which the quadratic equation $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are :
(A) $\frac{1}{2}, \frac{2}{9}$ (B) $0, \frac{1}{2}$ (C) $\frac{2}{9}$ (D) $0, \frac{1}{2}, \frac{2}{9}$

21. If the quadratic equation $2x^2 + ax + b = 0$ and $2x^2 + bx + a = 0$ $(a \neq 0)$ and $ax^2 - x + 1 = 0$ have a common root, the value of a + b is :
(A) – 3 (B) – 2 (C) – 1 (D) 0

22. If the equation $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root and $b \neq c$, then their other roots will satisfy the equation :
(A) $x^2 - (b + c)x + bc = 0$ (B) $x^2 - ax + bc = 0$
(C) $x^2 + ax + bc = 0$ (D) None of these

23. If both the roots of the equation $x^2 + mx + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ are common then :
(A) m = – 2 (B) m = – 1 (C) m = 0 (D) m = 1

24. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + ab = 0$ have one common root. The other roots of first and second equation are integers in the ratio 4 : 3. Then common root is : **[AIEEE-2008]**
(A) 1 (B) 4 (C) 3 (D) 2

Miscellaneous :

25. Solve $x^2 - 5x + 4 = 0$:
(A) 1 < x < 4 (B) – 4 < x < 1 (C) x < 1 and x > 4 (D) None of these

26. Solve $-x^2 + 6x - 8 = 0$:
(A) – 2 < x < 4 (B) – 4 < x < – 2 (C) 2 < x < 4 (D) None of these

27. For all $x \in R$, $x^2 + 2ax + 10 - 3a > 0$ then the interval in which 'a' lies is : **[IIT Screening-2004]**
(A) $(-\infty, -5)$ (B) (–5, 2) (C) $(5, \infty)$ (D) (2, 5)

28. The solution set contained in R of the inequation : $3^x + 3^{1-x} - 4 < 0$ is : **[EAMCET-2003]**
(A) (1, 3) (B) (0, 1) (C) (1, 2) (D) (0, 2)

29. The number of real solution of the equation $x^2 - 3|x| + 2 = 0$ is : **[AIEEE-2003]**
(A) 3 (B) 2 (C) 4 (D) 1

30. Product of real roots the equation $t^2x^2 + |x| + 9 = 0$: **[AIEEE-2002]**
(A) Is always positive (B) Is always negative (C) Does not exist (D) None of these

31. For the equation 3x2 + px + 3 = 0, p > 0. If one of the roots is square of the other, then p =
(A) $\frac{1}{2}3$ (B) 1 (C) 3 (D) $\frac{2}{3}$ **[IIT Screening-2000]**

32. The roots of the equation $|x^2 - x - 6| = x + 2$ are :
(A) – 2, 1, 4 (B) 0, 2, 4 (C) 0, 1, 4 (D) – 2, 2, 4

33. If α, β are the roots of $x^2 + x + 1 = 0$, the equation whose roots are $(\alpha^{19}, \beta^7$ is : **[IIT 1994]**
(A) $x^2 - x - 1 = 0$ (B) $x^2 - x + 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

34. The equation of the smallest degree with real coefficients having 1 + 1 as one of the roots is :
(A) $x^2 + x + 1 = 0$ (B) $x^2 - 2x + 2 = 0$ (C) $x^2 + 2x + 2 = 0$ (D) $x^2 + 2x - 2 = 0$
[Kerala Engineering -2002]

35. If a, b, c, d are positive reals such that a + b + c + d = 2 and M = (a + b) (c + d), then :
(A) $0 < M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$
[IIT Screening-2000]

36. Let a, b, c be real numbers such that 4a + 2b + c = 0 and ab > 0; then the quadratic equation $ax^2 + bx + c = 0$ has :
(A) Real roots (B) Non-real roots **[IIT 1990]**
(C) Purely imaginary roots (D) Only one real roots

37. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$, then the biquadratic P(x) Q(x) = 0 has :
(A) All the four roots real (B) No real roots **[IIT 1989]**
(C) Atleast imaginary roots (D) Two equal roots

38. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has : **[IIT 1989]**
(A) Two roots (B) Infinitely many roots(C) Only one roots (D) No root

39. Number of values of x satisfying the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$, where $t = x^2 - 2|x|$:
(A) 0 (B) 2 (C) 4 (D) 6

40. The of values of x which satisfy the expression : $(5 + 2\sqrt{6})^{x^2-3} + (5 - 2\sqrt{16})^{x^2-3} = 10$
(A) $\pm 2, \pm\sqrt{3}$ (B) $\pm\sqrt{2}, \pm 4$ (C) $\pm 2, \pm\sqrt{2}$ (D) $\pm\sqrt{2}, \pm\sqrt{3}$

41. If α and β are the roots of the equation $ax^2 + bx + c$, where (a,b,c) > 0, then α and β are :
(A) Rational numbers (B) Real and negative (C) Negative real parts (D) None of these

42. The number of quadratic equation which remain unchanged by squaring their roots, is :

(A) 0 (B) 2 (C) 4 (D) Infinitely many

43. If the equation $(\lambda^2+5\lambda+6)x^2+(\lambda^2-3\lambda+2)x+(\lambda^2-4)=0$ has more than two roots, then the value of λ is

(A) 2 (B) 3 (C) 1 (D) – 2

44. Find all the integral values of a for which the quadratic equation (x – a) (x – 10) + 1 = 0 has integral roots :

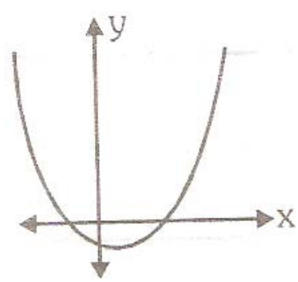
(A) 12, 8 (B) 4, 6 (C) 2, 0 (D) None of these

45. If one root of the quadratic equation $px^2 + qx + r = 0$ ($p \neq 0$) is a surd $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$ where p,q,r,a,b are all rationals then the other root is :

(A) $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$ (B) $a+\frac{\sqrt{a(a-b)}}{b}$ (C) $\frac{a+\sqrt{a(a-b)}}{b}$ (D) $\frac{\sqrt{a}-\sqrt{a-b}}{\sqrt{b}}$

46. Graph of $y = ax^2 + bx + c$ is given adjacently. What conclusions can be drawn from the graph :

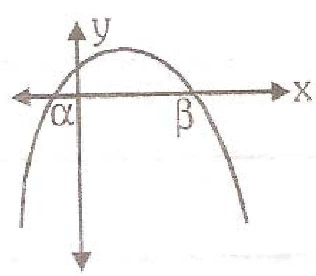
(i) $a > 0$ (ii) $b < 0$ (iii) $c < 0$ (iv) $b^2 - 4ac > 0$



(A) (i) and (iv) (B) (ii) and (iii) (C) (i), (ii) & (iv) (D) (i), (ii), (iii) & (iv)

47. The adjacently figure shows the graph of $y = ax^2 + bc + c$. Then which of the following is correct :

(i) $a > 0$ (ii) $b > 0$ (iii) $c > 0$ (iv) $b^2 < 4ac$



(A) (i) and (iv) (B) (ii) and (iii) (C) (iii) & (iv) (D) None of these

OBJECTIVE						ANSPER KEY							EXERCISE -6	
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	C	B	B	C	B	D	A	A	B	A	B	C	B	C	B
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	C	D	B	C	C	B	A	A	D	C	C	B	B	C	C
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	C	D	D	B	A	A	C	D	C	C	C	C	A	A	C
Que.	**46**	**47**													
Ans.	D	B													