# CO-ORDINATE GEOMETRY

## ★ INTRODUCTION

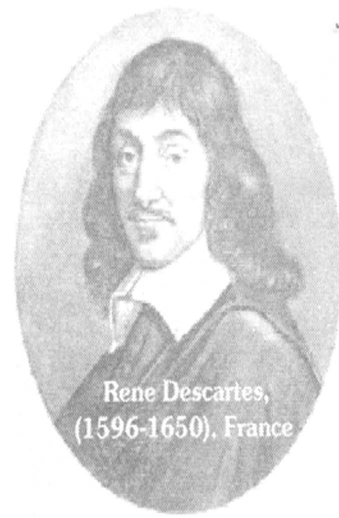
In the previous class, you have learnt to locate the position of a point in a coordinate plane of two dimensions, in terms of two coordinates. You have learnt that a linear equation in two variables, of the form ax + c = 0 (either $a \neq 0$ or $b \neq 0$) can be represented graphically as a straight line in the coordinate plane of x and y coordinates. In chapter 4, you have learnt that graph of a equation $ax^2 + bx + c = 0$, $a \neq 0$ is an upward parabola if $a > 0$ and a downward parabola if $a < 0$.

In this chapter, you will learn to find the distance between two given points in terms of their coordinates and also, the coordinates of the point which divides the line segment joining the two given points internally in the given ratio.

## ★ HISTORICAL FACTS

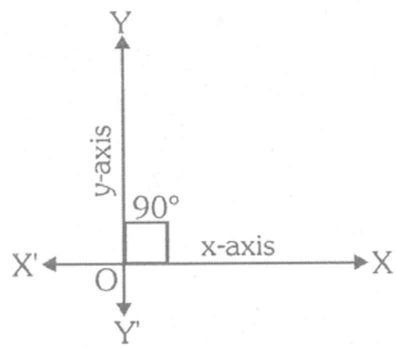
Rene Descartes (1596 - 1650), The 17th century French-Mathematician, was a thinker and a philosopher. He is called the father of Co-ordinate Geometry because he unified Algebra and Geometry which were earlier two distinct branches of Mathematics. Descartes explained that two numbers called co-ordinates are used to locate the position of a point in a plane.

He was the first Mathematician who unified Algebra and Geometry, so Analytical Geometry is also called Algebraic Geometry. Cartesian plane and Cartesian Product of sets have been named after the great Mathematician.




## ★ RECALL

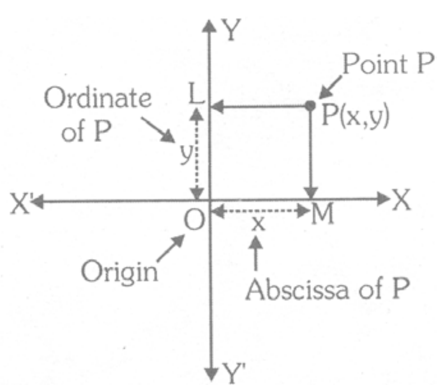
**Cartesian Co-ordinate system:**



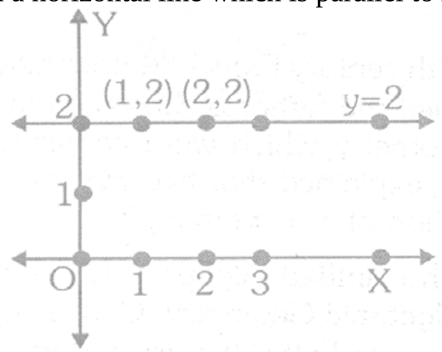

Let X'OX and Y'OY be two perpendicular straight lines intersecting each other at the point O. Then :

1. X'OX is called the x-axis or the axis of x.
2. Y'OY is called the y-axis or the axis of y.
3. The x-co-ordinate along OX is positive and along OX' negative, y-co-ordinate along OY (upward) is positive and along OY' (downward) is negative.
4. Both X'OX and Y'OY taken together in this order are called the rectangular axes because the angle between them is a right angle.
5. O is called the origin i.e., it is point of intersection of the axes of co-ordinates.

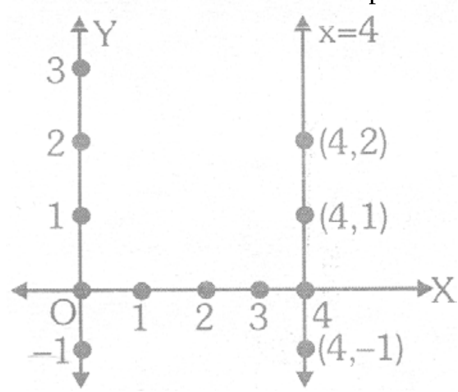
**Co-ordinates of a point :**



6. Abscissa of a point in the plane is its perpendicular distance with proper sign from y-axis.
7. Ordinate of a point in the plane is its perpendicular distance with proper sign from y-axis.
8. The y-co-ordinate any point on x-axis is zero.
9. The x-co-ordinate any point on x-axis is zero.
10. Any point in the xy-plane, whose y-co-ordinate is zero, lies on x-axis.
11. Any point in the xy-plane, whose x-co-ordinate is zero, lies on x-axis.
12. The origin has coordinates (0, 0).
13. The ordinates of all points on a horizontal line which is parallel to x-axis are equal i.e. y = constant = 2.

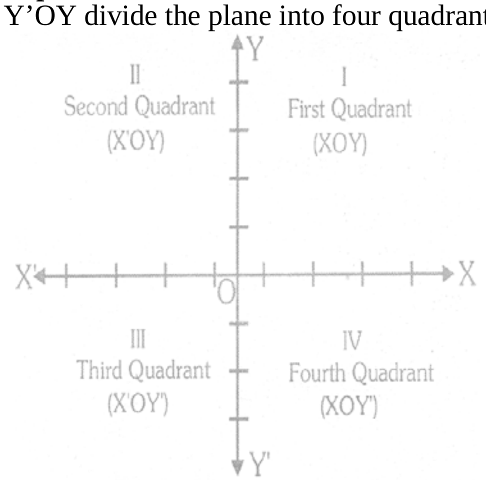



14. The abscissa of all points on a vertical line which is a line parallel to y-axis are equal i.e. x-constant = 4




**Four Quadrants of a Coordinate plane :**

The rectangular axes X'OX and Y'OY divide the plane into four quadrants as below :

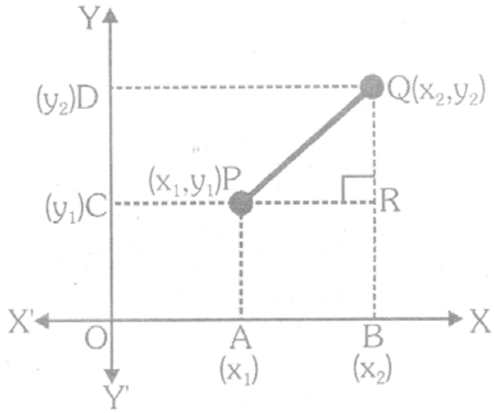



15. Any point in the I quadrant has (+ ve abscissa, + ve ordinate).
16. Any point in the II quadrant has (- ve abscissa, + ve ordinate).
17. Any point in the III quadrant has (- ve abscissa, - ve ordinate).
18. Any point in the IV quadrant has (+ ve abscissa, - ve ordinate).

## ★ DISTANCE FORMULA

> ***The distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ in a rectangular coordinate system is equal to $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$***

**Proof :** X,OX and Y'OY are the rectangular coordinate axes. P$(x_1, y_1)$ and Q $(x_2, y_2)$ are the given points. We draw PA and QB perpendiculars on the x-axis : PC and QD perpendicular on the y-axis,




Now, CP (produced) meets BQ in R and PR $\perp$ BQ.
We find PR = AB = OB – OA = $(x_2 - x_1)$
and QR = BQ – BR = BQ – AP = CD – OC = $(y_2 - y_1)$
In right $\Delta$PRQ, using Pythagoras theorem, we have

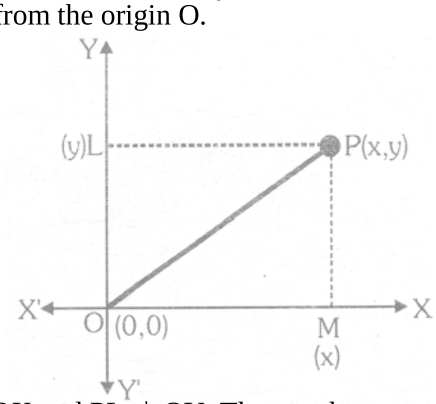
$$PQ^2 = PR^2 + QR^2 = (x_2-x_1)^2+(y_1-y_2)^2 \quad \Rightarrow \quad PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

We have taken the positive square roof value because distance between two points is a non-negative quantity.

**Distance of a Point from Origin :**

> **The distance of a point (x, y) form origin is $\sqrt{x^2+y^2}$ .**

**Proof :** Let us take a point P (x,y) in the given plane of axes X'OX and Y'OY as shown in the fig. Here, the point P (x, y) is in the first quadrant but it can be taken anywhere in all the four quadrants. We have to find the distance OP, i.e., the distance of the point P from the origin O.




Form the point P, draw PM $\perp$ OX and PL $\perp$ OY. Then we have
OM = x
MP = OL = y
$OP^2 = OM^2 + MO^2 = x^2 + y^2$

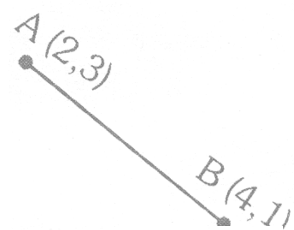
Therefore, OP = $\sqrt{x^2+y^2}$

**Test For Geometrical Figures :**

| | | | |
|---|---|---|---|
| (a) | For an isosceles | : | Prove that two sides are equal. |
| (b) | For an equilateral triangle | : | Prove that three sides are equal. |
| (c) | For a right-angled triangle | : | Prove that the sum of the squares of two sides is equal to the square of the third side. |
| (d) | For a square | : | Prove that all sides are equal and diagonals are equal. |
| (e) | For a rhombus | : | Prove that all sides are equal and diagonals are not equal. |
| (f) | For a rectangle | : | Prove that the opposites sides are equal and diagonals are also equal. |
| (g) | For a parallelogram | : | Prove that the opposite sides are equal in length and diagonals are not equal |

**Ex.1** Find the distance between the following pairs of points:
(a) (2,3), (4,1) (b) (-5, 7), (-1,3) (c) (a,b), (-1, - b) **[NCERT]**

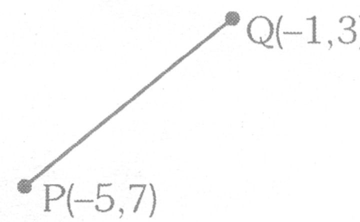
**Sol.** (a) The given points are : A (2,3), B (4,1).




Required distance $= AB = BA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(b) Distance between P(- 5, 7) and Q(-1, 3) is given by

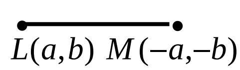



PQ = QP $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-1+5)^2+(3-7)^2} = \sqrt{16+6} = \sqrt{32}$

Required distance = PQ = QP = $4\sqrt{2}$ units

(c) Distance LM between L (a,b) and M (-a, -b) is given by




LM $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2} = \sqrt{4a^2=4b^2}$

$= \sqrt{4(a^2+b^2} = 2\sqrt{a^2+b^2}$ units

Ex.2 Find the points on x-axis which are at a distance of 5 units from the point A(-1, 4).

Sol. Let the point on x-axis be :P(x, 0).

Distance = PA = 5 units $\Rightarrow$ $PA^2 = 25$

$\Rightarrow$ $(x+1)^2+(0-4)^2 = 25$ $\Rightarrow$ $x^2+2x+1+16 = 25$

$\Rightarrow$ $x^2+2x+17 = 25$ $\Rightarrow$ $x^2+2x-8=0$

$$x = \frac{-2\pm\sqrt{(2)^2+4(8)}}{2} = \frac{-2\pm\sqrt{36}}{2} = \frac{-2\pm 6}{2}$$

$$= \frac{-2+6}{2}, \frac{-2-6}{2} = \frac{4}{2}, -\frac{8}{2} = 2, -4$$

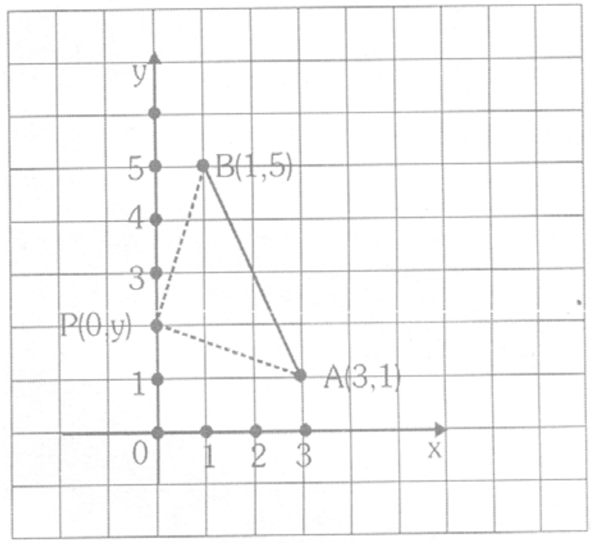
Required point on x-axis are (2, 0) and (-4, 0)

Verification: PA = $\sqrt{(2+1)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

PA = $\sqrt{(-4+1)^2+(0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

**Ex.3** What point on y-axis is equidistant from the points (3, 1) and (1, 5)

**Sol.** Since the required point P(say) is on the y-axis, its abscissa (x-co-ordinate) will be zero. Let the ordinate (y-co-ordinate) of the point be y.




Therefore co-ordinates of the point P are : (0, y)

i.e., P (0, y)

Let A and B denote the points (3, 1) and (1, 5) respectively.
PA = PB …(given)

Squaring we get :
$PA^2 = PB^2$

$\Rightarrow \quad (0-3)^2+(y-1)^2=(0-1)^2+(y-5)^2$

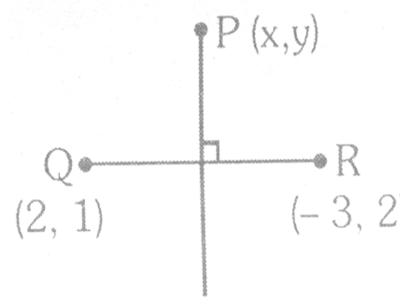
$\Rightarrow \quad 9+y^{2}1-2y=1+y^{2}+25-10y$

$\Rightarrow \quad y^2-2y+10-y^2-10y+26 \Rightarrow -2y+10y=26-10 \Rightarrow 8y=16 \Rightarrow y=2$

The required point on y-axis equidistant from A(3, 1) and B(1, 5) is P(0, 2).

**Ex.4** If Q(2, 1) and R(-3, 2) and P(x, y) lies on the right bisector of QR then show that 5x – y +

**Sol** Let P(x, y) be a point on the right bisector of QR :
Q(2, 1) and R(-3, 2) are equidistant from P(x, y), then we must have :
PQ = PR




$\Rightarrow \quad PQ^2 = PR^2$

$\Rightarrow \quad (x-2)^2+(y-1)^2=(x+3)^2+(y-2)^2$

$\Rightarrow \quad (x^2-4x+4)\;+(y^2-2y+1)\;=(x^2+6x+9)\;+(y^2-4x+4)$

$\Rightarrow \quad -4x-2y\;+5=6x-4y+13$

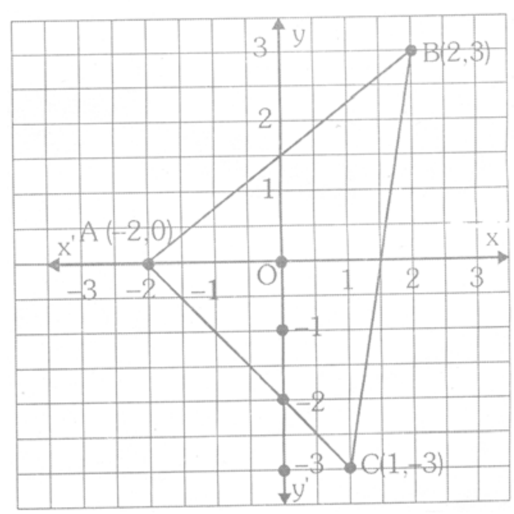
$\Rightarrow \quad 10x-2y-8=0$

$\Rightarrow \quad 2(5x-y+4)\;=0$

$\Rightarrow \quad 5x-y+4=0$

**Ex.5** The vertices of a triangle are (-2, 0), (2, 3) and (1, -3).
Is the triangle equilateral : isosceles or scalene ?

**Sol.** We denote the given point (-2, 0), (2, 3) and (1, -3) by A, B and C respectively then :




$A(-2,0), B(2,3), C(1,-3)$

$AB=\sqrt{(2+2)^2+(3-0)^2}=\sqrt{(4)^2+(3)^2}=5$

$BA=\sqrt{(1-2)^2+(-3-3)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{37}$

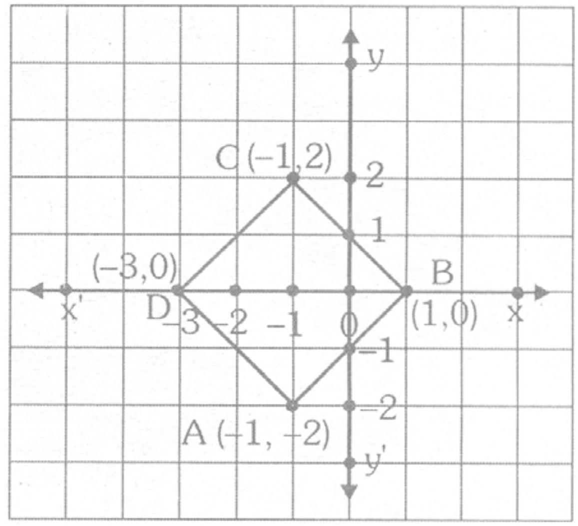
$BC = \sqrt{(-2-1)^2 + (0+3)^2} = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$

Thus we have $AB \neq BC \neq CA$

$\Rightarrow$ ABC is a scalene triangle

**Ex.6** Name the quadrilateral formed, if any, by the following points, and give reasons for your answer.
$(-1,-2), (1,0), (-1, 2), (-3,0)$ **[NCERT]**

**Sol.** A $(-1,-2), B(1,0) C(-1, 2), D(-3, 0)$




Determine distances : AB, BC, CD, DA, AC and BD.

AB = $\sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

AB = BC = CD = DA

The sides of the quadrilateral are equal …………(1)

AC = $\sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$

BD = $\sqrt{(-3+1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

Diagonal AC = Diagonal BD………(2)

From (1) and (2) we conclude that ABCD is a square.

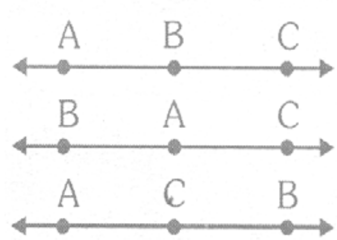
## ★ COLLINEARITY OF THREE POINTS

Let A, B and C three given points. Point A, B and C will be collinear, If the sum of lengths of any two line-
segments is equal to the length of the third line-segment.
In the adjoining fig. there are three point A, B and C.
Three point A, B and C are collinear if and only if

(i) AB + BC = AC

or (ii) AB + AC = BC

or (iii) AC + BC = AB




**Ex.7** Determine whether the points (1, 5) (2, 3) and (-2, -11) are collinear. **[NCERT]**

**Sol.** The given points are : A(1, 5), B(2, 3) and C(-2, -11).
Let us calculate the distance : AB, BC and CA by using distance formula.

AB = $\sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{(-2-2)^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+296} = \sqrt{265}$

From the above we see that : AB + BC $\neq$ CA

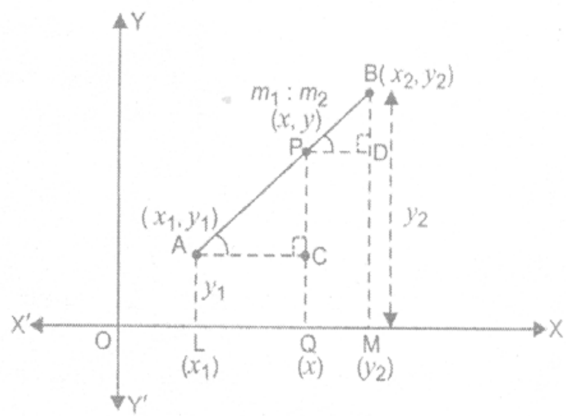
Hence the above stated points A(1, 5) B(2, 3) and C(-2, -11) are not collinear.

## ★ SECTION FORMULA

***Coordinates of the point, dividing the line-segment joining the points ($x_1$, $y_1$) and ($x_2$, $y_2$) internally in the ratio $m_1 : m_2$ are given by*** $\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

**Proof.** Let P (x, y) be the point dividing the line-segment joining A($x_1$, $y_1$) internally in the ratio $m_1 : m_2$. We draw the perpendiculars AL, BM and PQ on the x-axis from the points A, B and P respectively. L, M and Q are the points on the x-axis where three perpendiculars meet the x-axis.

We draw AC $\perp$ PQ and PD $\perp$ BM. Here AC $\|$ x-axis and PD $\|$ x-axis.




$\Rightarrow$ AC $\|$ PD ($\because$ AC and PD both $\|$ x-axis)

$\Rightarrow$ $\angle$PAC = $\angle$BPD

Thus, in $\Delta$ACP and $\Delta$PDB, we have

$\angle$PAC = $\angle$BPD

and $\angle$ACP = $\angle$PDB = $90^0$

Then by AA similarity criterion,

$\Delta$ACP ~ $\Delta$PDB

$$\Rightarrow \quad \frac{AC}{PD} = \frac{PC}{BD} = \frac{AP}{PB} = \frac{m_1}{m_2}$$

$$\Rightarrow \quad \frac{AC}{PD} = \frac{m_1}{m_2} \text{ ........(1)} \quad \text{and} \quad \frac{PC}{BD} = \frac{m_1}{m_2} \text{ .........(2)}$$

AC = LQ = OQ – OL = (x - $x_1$)

PD = QM = OM – OQ = ($x_2$ - x)

Putting in (1), we get

$$\frac{(x-x_1)}{(x_2-x)} = \frac{m_1}{m_2} \quad \Rightarrow \quad m_2x - m_2x_1 = m_1x_2 - m_1x$$

$$\Rightarrow \quad m_1x + m_2x = m_1x_2 - m_2x_1 \quad \Rightarrow \quad (m_1+m_2)x = m_1x_2 - m_1x_1$$

$$\Rightarrow \quad x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$$

Now, PC = PQ – CQ = PQ – AL = (y – $y_1$)

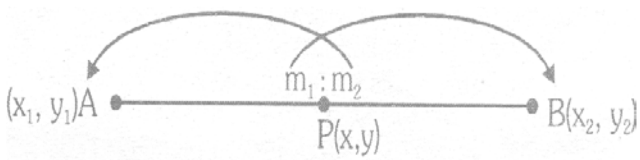
BD = BM – DM = BM – PQ = ($y_2$ – y)

Putting in (2), we get

$$\frac{y-y_1}{y_2-y} = \frac{m_1}{m_2} \quad \Rightarrow \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

Therefore, the coordinates of the point P are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

**Remark :** To remember the section formula, the diagram given below is helpful:




**Point Dividing a Line Segment in the Ratio k : 1 :**

If P$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ divides the line-segment, joining A($x_1$, $y_1$) and B($x_2$, $y_2$) internally in the ratio $m_1$ : $m_2$ we can express it as below :

P$\left(\frac{\frac{m_1}{m_2}x_2}{\frac{m_1}{m_2}+1}, \frac{\frac{m_1}{m_2}y_2+y_1}{\frac{m_1}{m_2}+1}\right)$ (By dividing the numerator and the denominator by $m_2$)

Putting $\frac{m_1}{m_2}$ = k, the ratio becomes k : 1 and the coordinates of P are expressed in the form

P $\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2y_1}{k+1}\right)$

Therefore, the coordinates of the point P, which divides the line-segment joining A ($x_1$, $y_1$) and B($x_2$, $y_2$) internally in the ratio k : 1, are given by $\left(\frac{kx_2+x1}{k+1}, \frac{ky_2+y1}{k+1}\right)$

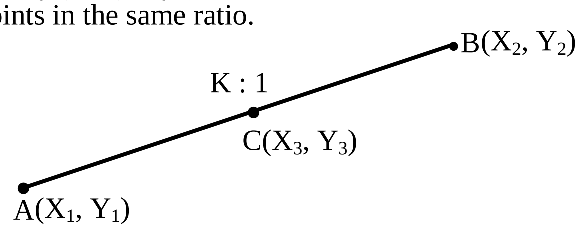
**Mid-point Formula :**

***Coordinates of the mid-point of the line-segment joining ($x_1$, $y_1$) and ($x_2$, $y_2$) are*** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$**.**

The mid-point M (x,y) of the line-segment joining A ($x_1$, $y_1$) and B($x_2$, $y_2$) divides the line-segment AB in the ratio 1 : 1. Putting $m_1 - m_2 = 1$in the section formula, we get the coordinates of the mid-point as $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

**Collinearity of three points :**

Three given points A($x_1$, $y_1$), B($x_2$, $y_2$), C($x_3$, $y_3$) are said to be collinear if one of them must divide the line segment joining the other two points in the same ratio.




Remark : Three points are called non-collinear if one of them divides the line segment joining the other two points in different ratios.

## COMPETITION WINDOW

### SECTION FORMULA FOR EXTERNAL DIVISION

The co-ordinates of line point which divides the line segment joining the point ($x_1$, $y_1$) and ($x_2$, $y_2$) externally in the ratio m : n are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

e.g., In the following case, C divides AB externally in ratio AC : BC

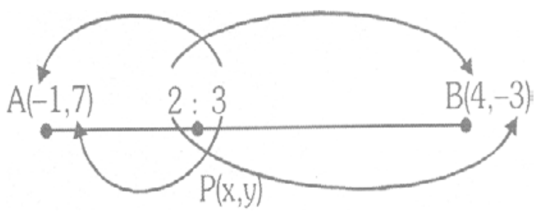
**A** **B** **C**

Let A = (1, 0), B = (4, 0) and 5 : 2 be the ratio in which C divides AB externally. Then co-ordinates of C are :

$$\left(\frac{5\times4-2\times1}{5-2}, \frac{5\times0-2\times0}{5-2}\right) = [6,0]$$

**Ex.8** Find the co-ordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3, **[NCERT]**

**Sol.** Let P(x, y) divides the line segment AB joining A(-1, 7) and B(4, -3) in the ratio 2 : 3. Ten by using section formula line the co-ordinates of P are given by :




$$\left(\frac{2\times4\times3\times(-1)}{2+3}, \frac{2\times(-3)+3\times7}{2+3}\right) = P\left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = P\left(\frac{5}{5}, \frac{15}{5}\right) = \text{P(1, 3)}$$

Hence the required point of division which divides the line segment joining A(-1, 7) and (4, -3) in the ratio 2 : 3 is P(1, 3).

**Sol.**
(-2, 2) (2, 8)
A P Q R B

It is given that AB is divided into four equal parts : AP = PQ = QR = RB

Q is the mid-point of AB, then co-ordinates of Q are : $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,5)$

P is the mid-point of AQ, then co-ordinates of P are : $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(\frac{-2}{2}, \frac{7}{2}\right) = \left(-1, \frac{7}{2}\right)$

Also, R is the mid-point of QB, then co-ordinates of R are : $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(\frac{2}{2}, \frac{13}{2}\right) = \left(-1, \frac{13}{2}\right)$

Hence, required co-ordinates of the points are :

$$P(-1\frac{7}{2}), Q(0, 5), R(1, \frac{13}{2})$$

**Ex.10** If the point C(-1, 2) divides the lines segment AB in the ratio 3 : 4, where the co-ordinates of A are (2, 5), find the coordinates of B.

**Sol.** Let C (-1, 2) divides the line joining A (2, 5) and B (x, y) in the ratio 3 : 4. Then.

$$C\left(\frac{3x+8}{7},\frac{3y+20}{7}\right)=C(-1,\ 2)$$

$\Rightarrow \quad \frac{3x+8}{7}=-1 \quad \& \quad \frac{3y+20}{7}=2$

$\Rightarrow \quad 3x+8=-7 \quad \& \quad 3y+20=14$

$\Rightarrow \quad x=-5 \quad \& \quad y=-2$

The coordinates of B are : B (-5, -2)

**Ex.11** Find the ratio in which the line segment joining the points (1, -7) and (6, 4) is divided by x-axis

**Sol.** Let C (x, 0) divides AB in the ratio k : 1.

By section formula, the coordinates of C are given by :

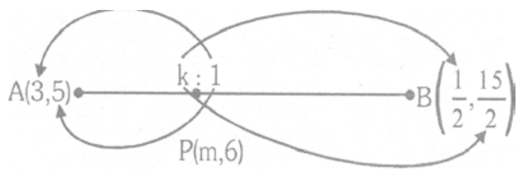
$$C\left(\frac{6k+1}{k+1},\frac{4k-7}{k+1}\right) \quad \Rightarrow \quad \frac{4k-7}{k+1}=0$$

$\Rightarrow \quad 4k-7=0 \Rightarrow k=\frac{7}{4}$

i,e., the x-axis divides AB in the ratio 7 : 4.

**Ex.12** Find the value of m for which coordinates (3,5), (m,6) and $\left(\frac{1}{2},\frac{15}{2}\right)$ are collinear.

**Sol.** Let P (m, 6) divides the line segment AB joining A (3,5) B $\left(\frac{1}{2},\frac{15}{2}\right)$ in the ratio k : 1.




Applying section formula, we get the co-ordinates of P : $\left(\frac{\frac{1}{2}k+3\times1}{k+1},\frac{\frac{15}{2}k+5\times1}{k+1}\right)=\left(\frac{k+6}{2(k+1)},\frac{15k+10}{2(k+1)}\right)$

But P (m, 6) = P$\left(\frac{k+6}{2(k+1)},\frac{15k+10}{2(k+1)}\right) \quad \Rightarrow \quad m=\frac{k+6}{2(k+)}$ and also $\frac{15k+10}{2(k+1)}=6$

$\Rightarrow \quad \frac{15k+10}{2(k+1)}=6 \quad \Rightarrow \quad 15k+10=12(k+1)$

$\Rightarrow \quad 15k+10=12k+12 \quad \Rightarrow \quad 15k-12k=12-10$

$\Rightarrow \quad 3k = 2 \quad \Rightarrow \quad k=\frac{2}{3}$

Putting $k=\frac{2}{3}$ in the equation $m=\frac{k+6}{2(k+)}$ we get :

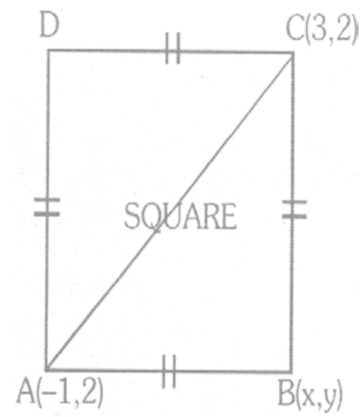
$$m=\frac{\left(\frac{2}{3}+6\right)}{2\left(\frac{2}{3}+1\right)}=\frac{\left(\frac{2+18}{3}\right)}{2\left(\frac{2+3}{3}\right)}=\frac{20}{3}\times\frac{3}{10}=\frac{20}{10} \qquad \left(\because k=\frac{2}{3}\right)$$

$$m=\frac{10\times2}{10}=2$$

Required value of m is 2 $\Rightarrow$ m = 2

**Ex.13** The two opposite vertices of a square are (-1, 2) and (3, 2). Find the co-ordinates of the other two vertices.

**Sol.** Let ABCD be a square and two opposite vertices of it are A(-1, 2) and C(3, 2) ABCD is a square.




$\Rightarrow$ AB = BC $\Rightarrow$ $AB^2 = BC^2$

$\Rightarrow$ $(x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow$ $x^2+2x+1=x^2-6x+9$ $\Rightarrow$ $2x\ +6x=9-1=8$

$\Rightarrow$ $8x=8\Rightarrow x=1$

ABCD is right $\Delta$ at B, then

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow$ $(3+1)^2+(2-2)^2=(x+)^2+(y-2)^2+(x-3)^2=(y-2)^2$

$\Rightarrow$ $16=2(y-2)^2+(1+1)^2+(1-3)^2$

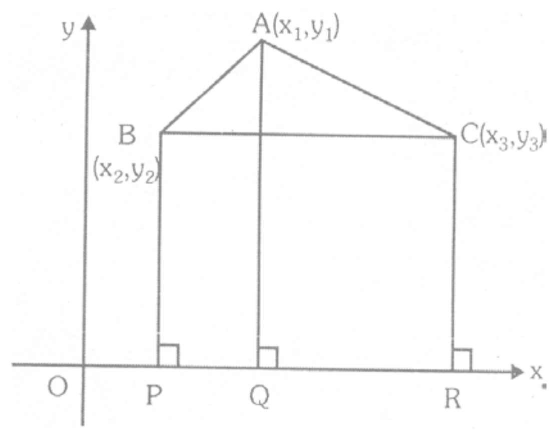
$\Rightarrow$ $16=\ 2(y-2)^2+4+4\Rightarrow 2(y-2)^2\ =16-8=8$

$\Rightarrow$ $(y-2)^2=4$ $\Rightarrow$ $y-2=\pm 2\Rightarrow$ y = 4 and 0

i.e., when x = 1 then y = 4 and 0

Co-ordinates of the opposite vertices are : B(1, 0) or D(1, 4)

## ★ AREA OF A TRIANGLE




In your pervious classes, you have learnt to find the area of a triangle in terms of its base and corresponding altitude as below :

Area of triangle = $\frac{1}{2}$ $\times$ base $\times$ altitude.

In case, we know the lengths of the three sides of a triangle, then the area of the triangle can be obtained by using the Heron's formula.

In this section, we will find the area of a triangle when the coordinates of its three vertices are given. The lengths of the three sides can be obtained by using distance formula but we will not prefer the use of Heron's formula.

Some times, the lengths of the sides are obtained as irrational numbers and the application of Heron's formula becomes tedious. Let us develop some easier way to find the area of a triangle when the coordinates of its vertices are given.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the given three points. Through A draw AQ $\perp$ OX, through B draw BP $\perp$ OX and through C draw CR $\perp$ OX.

Form the fig. AQ = $y_1$, BP = $y_2$ and CR = $y_3$, OP = $x_2$, OQ = $x_1$ and OR = $x_3$

$\Rightarrow$ PQ = $x_1 - x_2$ ; QR = $x_3 - x_1$ and PR = $x_3 - x_2$

Area of trapezium = $\frac{1}{2}$ (sum of parallel side) $\times$ distance between parallel lines

ar. ($\Delta$ ABC) = ar.(Trap.ABCD) + ar. (Trap.AQRC) – ar. (Trap. BPRC)

$= \frac{1}{2}(BP + AQ) \times PQ + \frac{1}{2}(AQ + CR) \times QR - \frac{1}{2}(PB + CR) \times PR$

$= \frac{1}{2}(y_2+y_1)(x_1-x_2)+\frac{1}{2}(y_2+y_3)(x_3-x_1)-\frac{1}{2}(y_2+y_3)(x_3-x_2)$

$= \frac{1}{2}\left|x_1(y_2+y_1-y_1-y_3)+x_2(y_2+y_3-y_2-y_1)+x_3(y_1+y_3-y_2-y_3)\right|$

$= \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

Area of $\Delta$ABC = $\frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

**Condition of collinearity of three points :**
The given points A($x_1$,$y_1$), B($x_2$,$y_2$) and C($x_3$,$y_3$) will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of $\Delta$ABC = 0

$$\Rightarrow \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| = 0$$

$$\Rightarrow \boxed{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0}$$

is the required condition for three points to be collinear.

**Ex.14** The co-ordinates of the $\Delta$ABC are A(4, 1), D(3, 2) and C(0, K). Given that the area of $\Delta$ABC is 12 unit$^2$. Find the value of k.

**Sol.** Area of $\Delta$ABC formed by the given-points A(4, 1), B(-3, 2) and C(0, k) is

$= \frac{1}{2}|4(2-k)+(-3)(k-1)+0(1-2)| = \frac{1}{2}|18-4k-3k+3| = \frac{1}{2}(11-7k)$

But area of $\Delta$ABC = 12 unit$^2$ ……..(given)

$\frac{1}{2}| = \frac{1}{2}(11-7k)| = 24 \quad \Rightarrow \quad |(11-7k)| = 24$

$\pm(11-7k) = 24 \quad \Rightarrow \quad 11-7k = 24 \, or -(11-7k) = 24$

$-7k = 24-11 = 13 \quad \Rightarrow \quad k = -\frac{13}{7}$

or $-(11-7k) = 24 \quad \Rightarrow \quad -11+7k = 24$

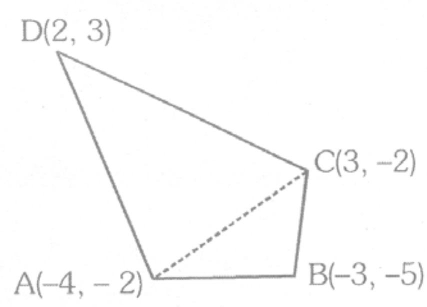
$\Rightarrow 7k = 24+11 = 35 \quad \Rightarrow \quad k = \frac{35}{7} = 5$

**Ex.15** Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2), and (2, 3).

**[NCERT]**

**Sol.** Join A and C

The given points are A(-4, -2), B(-4, -2), C(-4, -2) and D(2, 3)




Area of $\Delta$ABC

$= \frac{1}{2}|(-4)(-5+2)-3(-2+2)+3(-2+5)|$

$= \frac{1}{2}|20-8-6+15| \qquad = \frac{21}{2} - 10.5$ sq. units

Area of $\Delta$ACD

$= \frac{1}{2}|(-4)(-2-3)+3(-2+2)+3(-2+5)|$

$= \frac{1}{2}|\ 20+15| = \frac{35}{2} = 17.5$ sq. units

Area of quadrilateral ABCD = ar. ($\Delta$ABC) + ar. ($\Delta$ACD) = (10.5 + 17.5) sq. units = 28 sq. units

**Ex.16** Find the value of p for which the points (-1, 3), (2, p), (5, -1) are collinear.

**Sol.** The given points A (1, 3), B(2, p), C(5, -1) are collinear.

$\Rightarrow$ Area of $\Delta$ABC formed by these points should be zero.
$\Rightarrow$ The area of $\Delta$ABC = 0
$\Rightarrow$ $\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| = 0$
$\Rightarrow$ $-1(p+1)+2(-1-3)+5(3-p)=0$
$\Rightarrow$ $-p-1-8+15-5p=0$
$\Rightarrow$ $-6p+15-9=0 \Rightarrow 6p=-6 \Rightarrow p=1$

Hence the value of p is 1.

## COMPETITION WINDOW

### AREA OF A QUADRILATERAL

If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(x_4, y_4)$ are vertices of a quadrilateral, its area

$$\frac{1}{2}|\ (x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3)+(x_4y_1-x_1y_4)|$$

### AREA OF A POLYEON

If $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots\ldots (x_n, y_n)$ are vertices of a polygon of n sides, its area

$$\frac{1}{2}|\ (x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_4-x_4y_3)+\ldots.+(x_ny_1-x_1y_n)|$$

**Remark :** (i) If the area of a quadrilateral joining the four points is zero, the four points are collinear.
(ii) If two opposite vertex of a square are A($x_1$, $y_1$) and C($x_2$, $y_2$) then it's area is

$$\frac{1}{2}[(x_2-x_1)^2+(y_2-y_1)^2]$$

### TRY OUT THE FOLLOWING

(i) Find the area of the quadrilateral formed by joining the four points (1, 1), (3, 4), (5, -2) & (4, -7).
(ii) Find the area of the pentagon whose vertices are A(1, 1), B(7, 21), C(7, -3) D(4, -7) and E(0, -3).
(iii) If the Co-ordinates of two opposite vertex of a square are (a, b) and (b, a), find the area of the square.

### ANSWERS

(i) $\frac{41}{2}$ sq. nuits (ii) $\frac{137}{2}$ sq. nuits (i) $(a-b)^2$ sq. nuits

★ **SYNOPSIS**

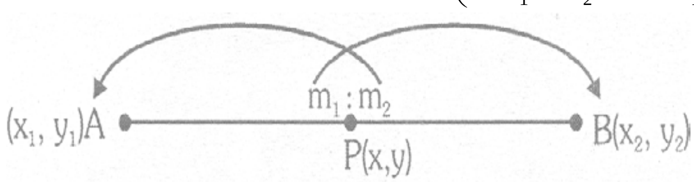
⏩ ***Distance Formula : The distance between two points ($x_1$, $y_2$) and ($x_2$, $y_2$) in a rectangular coordinate system is equal to*** $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ ***. The distance of a point (x, y) from origin is*** $\sqrt{x^2+y^2}$

**Test For Geometrical Figures :**

| | | | |
|---|---|---|---|
| (a) | For an isosceles triangle | : | Prove that at least two sides are equal |
| (b) | For an equilateral triangle | : | Prove that three sides are equal |
| (c) | For a right-angled triangle | : | Prove that the sum of the squares of two sides is equal to the square of the third side. |
| (d) | For a square | : | Prove that all sides are equal and diagonals are equal. |
| (e) | For a rhombus | : | Prove that all sides are equal and diagonals are not equal. |
| (f) | For a rectangle | : | Prove that the opposite sides are equal and diagonals are also equal. |
| (g) | For a parallelogram | : | Prove that the opposite sides are equal in length and diagonals are not equal. |

⏩ **Collinearity of three points :** Let A, B and C there given points. Point A, B and C will be collinear if the sum, of lengths of any two line-segment is equal to the length of the third line-segment.
In the adjoining fig. there are three point A, B and C.
Three points A, B and C are collinear if and only if
(i) AB + BC = AC or (ii) AB + AC = BC or (iii) AC + BC = AB

⏩ **Section Formula : Coordinates of the point, dividing the line-segment joining the points ($x_1$, $y_1$) and ($x_2$, $y_2$) internally in the ratio : $m_1$ $m_2$ are given by** $\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$




**Mid-point Formula : Coordinates of the mid-point of the line-segment joining ($x_1$, $y_1$) and ($x_2$, $y_2$) are**
$\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

⏩ Area of triangle : $\boxed{\text{Area of } \Delta ABC = \dfrac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+y_3(y_1-y_2)\right|}$

**Condition of collinearity of three points :** The given points A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) will be collinear if the area of the triangle formed by them must be zero because triangle can not be formed.

$\Rightarrow$ area of $\Delta ABC = 0$

$\Rightarrow$ $\dfrac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+y_3(y_1-y_2)\right| = 0 \Rightarrow \boxed{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0}$

is the required condition for three points to be collinear.

## EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

### OBJECTIVE TYPE QUESTIONS

### CHOOSE THE CORRECT OPTION IN EACH OF THE FOLLOWING

**1.** The distance between the points (a, b) and (-a, - b) is :
(A) $a^2+b^2$ (B) $\sqrt{a^2+b^2}$ (C) 0 (D) $2\sqrt{a^2+b^2}$

**2.** The distance between points (a + b, b + c) and (a – b, c - b) is :
(A) $2\sqrt{a^2+b^2}$ (B) $2\sqrt{a^2+c^2}$ (C) $2.\sqrt{2b}$ (D) $\sqrt{a^2-c^2}$

**3.** The distance between points A(1, 3) and B(x, 7) is 5. The value of x > 0 is :
(A) 4 (B) 2 (C) 1 (D) 3.

**4.** The distance between the points (a cos $20^0$ + b sin $20^0$,0) and (a sin $20^0$ - b cos $20^0$) is :
(A) (a + b) (B) (a – b) (C) $\sqrt{a^2-b^2}$ (D) $\sqrt{a^2+b^2}$

**5.** Mid-point of the line-segment joining the points (-5, 4) and (9, - 8) is :
(A) (–7, 6) (B) (2, – 2) (C) (7, – 6) (D) (–2, 2).

**6.** The co-ordinates of the points which divides the join of (–2, 2) and (–5, 7) in the ratio 2 : 1 is :
(A) (4, – 4) (B) (–3, 1) (C) (–4, 4) (D) (1, –3).

**7.** The co-ordinates of the points on x-axis which is equidistant from the points (5, 4) and (–2, 3) are :
(A) (2, 0) (B) (3, 0) (C) (0, 2) (D) (0, 3).

**8.** The co-ordinates of the points on y-axis which is equidistant from the points (3, 1) and (1, 5) are :
(A) (0, 4) (B) (0, 2) (C) (4, 0) (D) (2, 0).

**9.** The coordinates of the centre of a circle are (– 6, 1.5). If the ends of a diameter are (– 3, y) and (x, - 2) then:
(A) x = - 9, y = 5 (B) x = 5, y = - 9 (C) x = 9, y = 5 (D) None of these

**10.** The points (- 2, 2), (8, - 2) and (-4, - 3) are the vertices of a :
(A) equilateral $\Delta$ (B) isosceles $\Delta$ (C) right $\Delta$ (D) None of these

**11.** The points (1, 7), (4, 2) (- 1, 1) (- 4, 4) are the vertices of a :
(A) parallelogram (B) rhombus (C) rectangle (D) square.

**12.** The line segment joining (2, - 3) and (5, 6) is divided by x-axis in the ratio:
(A) 2 : 1 (B) 3 : 1 (C) 1 : 2 (D) 1 : 3.

**13.** The line segment joining the points (3, 5) and (- 4, 2) is divided by y-axis in the ratio:
(A) 5 : 3 (B) 3 : 5 (C) 4 : 3 (D) 3 : 4.

**14.** If (3, 2), (4, k) and (5, 3) are collinear then k is equal to :

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{3}{5}$

**15.** If the points (p, 0), (0, q) and (1, 1) are collinear then $\frac{1}{p}+\frac{1}{q}$ is equal to :

(A) – 1 (B) 1 (C) 2 (D) 0

**16.** Two vertices of a triangle are (-2, -3) and (4, -1) and centroid is at the origin. The coordinates of the third vertex of the triangle are :

(A) (– 2, 3) (B) (–3, – 2) (C) (–2, 4) (D) (4, –2)

**17.** A (5, 1), B(1, 5) and C(–3, –1) are the vertices of $\Delta$ABC. The length of its median AD is :

(A) $\sqrt{34}$ (B) $\sqrt{35}$ (C) $\sqrt{37}$ (D) 6

**18.** Three consecutive vertices of a parallelogram are (1, –2), (3, 6) and (5, 10). The coordinates of the fourth vertex are :

(A) (–3, 2) (B) (2,– 3) (C) (3, 2) (D) (– 2, –3)

**19.** The vertices of a parallelogram are (3, –2), (4, 0), (6,– 3) and (5,– 5). The diagonals intersect at the point M. The coordinates of the point M are :

(A) $\left(\frac{9}{2},\frac{5}{2}\right)$ (B) $\left(\frac{7}{2},\frac{5}{2}\right)$ (C) $\left(\frac{7}{2},\frac{3}{2}\right)$ (D) None of these

**20.** If two vertices of a parallelogram are (3, 2) and (–1, 0) and the diagonals intersect at (2, –5), then the other two vertex are :

(A) (1,– 10), (5, –12) (B) (1,– 12), (5, –10) (C) (2, – 10) (D) (1, – 10), (2, – 12)

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| OBJECTIVE | | | | ANSWER KEY | | | EXERCISE-4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Que.** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| Ans. | D | C | A | C | B | C | A | B | A | C |
| **Que.** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| Ans. | D | A | D | C | C | C | C | C | A | B |

## EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

## SUBJECTIVE TYPE QUESTIONS

### SHORT ANSWER TYPE QUESTIONS

**1.** Find the distance between the points A and B in the following :

(i) $A(a+b, b-a), B(a-b, a+b)$ (ii) $A(1,-1), B(-\frac{1}{2}, \frac{1}{2})$

**2.** Find the distance between the points A and B in the following :

(i) $A(8-2), B(3-6)$ (ii) $A(a+b, a-b), B(a-b, -a-b)$

**3.** A point P lies on the x-axis and has abscissa 5 and a point Q lies on y-axis and has ordinate – 12. Find the distance PQ.

**4.** Find a relation between x and y such that the point (x, y) is equidistant from (7, 1) and (3, 5).

**5.** Using distance formula, show that the points A, B and C are collinear.

(i) $A(-1,-1), B(2,3), C(8,11)$ (ii) $A(-4,-2), B(-1,1), C(1, 3)$

**6.** Find a point on the x-axis which is equidistant from the points (5, 4) and (-2, 3).

**7.** Find a point on the x-axis which is equidistant from the points (-3, 4) and (2, 3).

**8.** Find the value of k, if the point (2, 3) is equidistant from the points A(k, 1) and B(7, k).

**9.** Find the value of k for which the distance between the point A(3k, 4) and B(2, k) is $5\sqrt{2}$ units.

**10.** Find the co-ordinates of the point which divides the line segment joining the points (1, -3) and (-3, 9) in the ratio 1 : 3 internally.

**11.** Find the mid-point of AB where A and B are the points (-5, 11) and (7, 3) respectively.

**12.** The mid-point of a line segment is (5, 8). If one end points is (3, 5), find the second end point.

**13.** The vertices of a triangle are A(3, 4), (7, 2) and C(-2, -5). Find the length of the median through the vertex A.

**14.** The co-ordinates of A and B are (1, 2) and (2, 3) respectively. Find the co-ordinates of R on line segment AB so that $\frac{AR}{RB} = \frac{4}{3}$.

**15.** Find the co-ordinates of the centre of a circle, the co-ordinates of the end points of a diameter being (-3, 8) and (5, 6)

**16.** Find the co-ordinates of the point where the diagonals of the parallelogram formed by joining the points (-2, -1), (1, 0), (4, 3) and (1, 2) meet.

**17.** Find the ratio in which the line segment joining the points (3, 5) and (-4, 2) is divided by y-axis.

**18.** In what ratio in does the point $\left(\frac{1}{2}, \frac{-3}{2}\right)$ divide the line segment joining the points (3, 5) and (-7, 9) ?

**19.** By using section formula, show that the points (-1, 2), (2, 5) and (5, 8) are collinear.

**20.** Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

**21.** Show that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

**22.** Find the area of the triangle whose vertices are (3, 2) (-2, -3) and (2, 3).

**23.** For what value of m, the points (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear ?

### LONG ANSWER TYPE QUESTION

**1.** Prove that the points (1, 4), (3, 6) and (9, -2) are the vertices of an isosceles triangle.

**2.** Find the co-ordinates of the point equidistant from three given points A(5, 1), B(-3, -7) and C(7, -1).

**3.** Show that the points (7, 10), (-2, 5) and (3, - 4) are the vertices of an isosceles right triangle.

**4.** Prove that the points (0, 1), (1, 4), (4, 3) and (3, 0) are the vertices of a square.

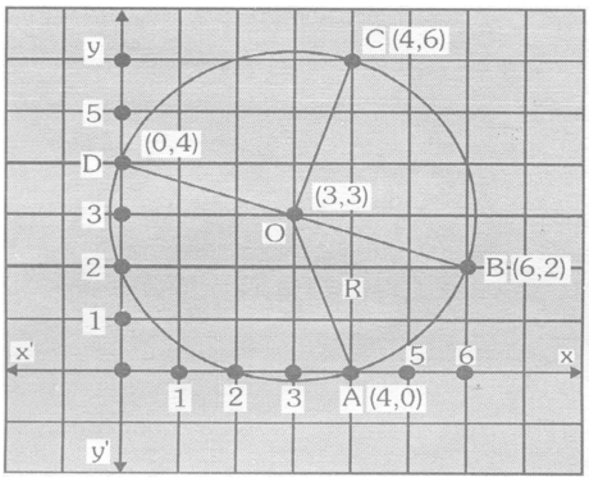
**5.** Prove that the points (-4,- 1), (-2,- 4), (4, 0) and (2, 3) are the vertices of a rectangle.

**6.** If two vertices of an equilateral triangle are (0, 0) and $(3, \sqrt{3})$, find the third vertex of the triangle.

**7.** A (3, 4) and C (1, -1) are the two opposite angular points of a square ABCD. Find the co-ordinates two vertices

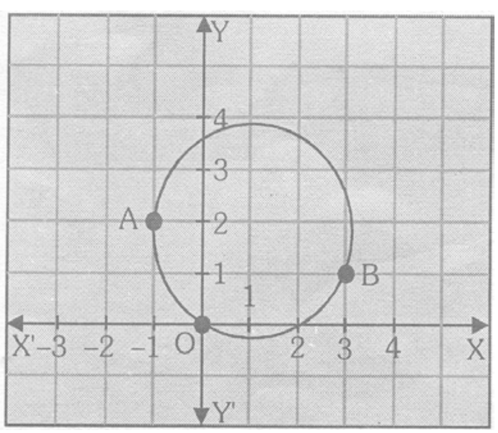
**8.** Find the co-ordinates of the point equidistant from the point A(-2, -3), B(-1, 0) and C(7, -6).

**9.** Show that (3, 3) is the centre of the circle passing through the points (4, 6), (0, 4), (6, 2) and (4, 0). What radius of the circle.




**10.** If A (2, -1), B(3, 4), C(-2, 3) and D (-3, -2) be four points in a co-ordinates plane, show that ABCD is a rhombus but not a square. Find the area of the rhombus.

**11.** In figure, find the co-ordinates of the centre of the circle which is drawn through the points A, B and O.



**12.** The line segment joining the points (3, -1) and (1, 2) is trisected at the points P and Q. If the co-ordinates of P and Q are (p, -2) and $\left(\frac{5}{2}, q\right)$ respectively, find the values of P and Q.

**13.** What will be the value of y if the point $\left(\frac{23}{5}, y\right)$, divides the line segment joining the points (5, 7) and (4, 5) in the ratio 2 : 3 internally.

**14.** Find the co-ordinates of the points which divide the line segment joining the points (-4, 0) and (0, 6) in 4 equal parts.

**15.** If the points (10, 5) (8, 4) and (6, 6) are the mid-points of the sides of a triangle, find its vertices.

**16.** Find the area of the quadrilateral ABCD formed by the points A (–2, –2), B (5, 1), C (2, 4) and (–1, 5)..

**17.** Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, – 3). Hence, find the area of the triangle formed by these points.

**18.** A (4, 3), B (6, 5) and C (5, – 2) are the vertices of $\Delta$ABC.

(i) Find the co-ordinates of the centroid G of $\Delta$ABC. Find the area of $\Delta$ABC and compare it with area of $\Delta$ABC.

(ii) If D is the mid-point of BC, find the co-ordinates of D. Find the co-ordinates of a point P on AD such that AP : PD = 2 : 3. Find the area of $\Delta$ABC and compare it with area of $\Delta$ABC.

**19.** ABCDE is a polygon whose vertices are A(–1, 0), B(4, 0), C(4, 4), D(0, 7) and E(–6, 2). Find the area of the polygon.

**20.** Name the quadrilateral formed by joining the points (1, 2), (5, 4), (3, 8) and (–1, 6) in order. Find also the area of the region formed by joining the mid-points of the sides of this quadrilateral.

| CO-ORDINATE GEOMETRY | ANSWER KEY | EXERCISE-2 (X)- |
|---|---|---|

**SHORT ANSWER TYPE QUESTION :**

**1. (i)** $2\sqrt{a^2+b^2}$ units, **(ii)** $\frac{3\sqrt{2}}{2}$ units **2. (i)** $\sqrt{41}$ units, **(ii)** $2\sqrt{a^2+b^2}$ units **3.** 13 units

**4.** x – y = 2 **6.** (2, 0) **7.** (0, 6) **8.** k = 13 **9.** k = –1 or k = 3 **10.** (0, 0) **11.** (1, 7) **12.** (7, 11)

**13.** $\frac{\sqrt{122}}{2}$ units, **14.** $\left(\frac{11}{7}, \frac{18}{7}\right)$ **15.** (1, 7) **16.** (1, 1) **17.** 3 : 4 **18.** 1 : 3 **20.** 5 units **22.** 5 sq. unit

**23.** m = 2

**LONG ANSWER TYPE QUESTIONS :**

**1.** (2, – 4) **2.** $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$ 3. $\left(\frac{9}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, \frac{5}{2}\right)$ **4.** (3, – 3) **5.** $\sqrt{10}$ units **6.** 24 sq. units

**7.** $\left(\frac{15}{14}, \frac{25}{14}\right)$ **8.** $p = \frac{7}{3}, q = 0$ **9.** $\frac{31}{5}$ **10.** $\left(-3, \frac{3}{2}\right), (-2, 3), \left(-1, \frac{9}{2}\right)$ **11.** (8, 7), (12, 3), (4, 5)

**16.** 26 sq. units **17.** (–2, 0), 10 sq. units

**18.** **(i)** G (5, 2) ; ar ($\Delta$GBC) = 2 sq. units ; ar ($\Delta$GBC) : ar ($\Delta$ABC) = 1 : 3

**(ii)** $D\left(\frac{11}{2}, \frac{3}{2}\right); P\left(\frac{23}{5}, \frac{12}{5}\right)$ ; ar ($\Delta$PBC) = $\frac{18}{5}$ sq. units ; ar ($\Delta$PBC) : ar ($\Delta$ABC) = 3 : 5

**19.** 44 sq. units **20.** Square ; 10 sq. units.

## EXERCISE – 3 (FOR SCHOOL/BOARD EXAMS)

### PREVIOUS YEARS BOARD (CBSE) QUESTIONS

**1.** Show that the point A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are the vertices of a square. **[Delhi-2004]**

**2.** Determine the ratio in which the point P(m, 6) divide the join of A(–4, 3) and B(2, 8). Also find the value of m.

**OR**

A(3, 2) and B(–2, 1) are two vertices of a triangle ABC, whose centroid G has coordinates $\left(\frac{5}{3},-\frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle. **[Delhi-2004]**

**3.** Show that the points A(2, –2), B(14, 10), C(11, 13) and D(–1, 1) are the vertices of a rectangle. **[Al-2004]**

**4.** Prove that the coordinates of the centroid of a $\Delta$ABC, with vertices. A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) are given by $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+_3}{2}\right)$ **[Al-2004]**

**5.** Determine the ratio in which the point (–6, a) divide the join of A(–3, –1) and B(–8, 9). Also find the value of a. **[Al-2004]**

**6.** Find the point on the x-axis which is equidistant from the points (–2, 5) and (2, –3) **[Al-2004]**

**7.** Prove that the points A(0, 1), B(1, 4), C(4, 3) and (3, 0) are the vertices of a square. **[Foreign-2004]**

**8.** Determine the ratio in which the point (a, –2) divide the join of A(–4, 3) and B(2, –4). Also find the value of a. **[Foreign-2004]**

**9.** Determine the ratio in which the point P(k, 2) divide the join of A(–3, 5) and B(5, 1). Also find the value of k. **[Foreign-2004]**

**10.** Determine the ratio in which the point P(b, 1) divide the join of A(7, –2) and B(–5, 6). Also find the value of b. **[Foreign-2004]**

**11.** The coordinates of the mid-point of the line joining the point (3p, 4) and (–2, 2q) are (5, p). Find the coordinates of p and q. **[Delhi-2004C]**

**12.** Two vertices of a triangle are (1, 2) and (3, 5). If the controid of the triangle is at the origin, find the coordinates of the third vertex.

**OR**

If 'a' is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin, find the coordinates of the vertices of the triangle ABC. **[Delhi-2004C]**

**13.** Find the ratio in which the one-segment joining the points (6, 4) and (1, –7 ) is divided by x-axis . **[Al-2004C]**

**OR**

The coordinates of two vertices A and B of a tnangle An are (1, 4) and (5, 3) respectively. If the coordinates of the centroid of $\Delta$ABC are (3, 3), find the coordinates of the third vertex C. **[Al-2004C]**

**14.** Find the value of m for which the points with coordinates (3, 5), (m, 6) and $\left(\frac{1}{2},\frac{15}{2}\right)$ are collinear. **[Al-2004C]**

**15.** Find the value of x such that PQ = QR where the coordinates of P,Q and R are (6,–1);(1,3) and (x,8)respectively.

**OR**

Find a point on x-axis which is equidistant from the points (7, 6) and (–3, 4). **[Delhi-2004]**

**16.** The line-segment joining the points (3, –4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, 2) and $\left(\frac{5}{3}, q\right)$ respectively, find the values of p and q. **[Delhi-2005]**

**17.** Prove that the points (0, 0), (5, 5) are vertices of a right isosceles triangle.

**OR**

If the point P(x, y) is equidistant from the point A(5, 1) and B(–1, 5), prove that 3x = 2y. **[Al-2005]**

**18.** The line joining the points (2, 1) and (5, –8) is trisected at the points p and Q. If point P lies on the line 2x – y + k = 0, find the value of k. **[Al-2005]**

**19.** Show that the points (0, –1); (2, 1); (0, 3) and (–2, 1) are the vertices of a square.

**OR**

Find the value of K such that the point (0, 2) is equidistant from the points (3, K) and (K, 5). **[Foreign-2005]**

**20.** The base BC of an equilateral $\Delta$ABC lies on y-axis. The coordinates of point C are (0, –3). If the origin is the mid-point of the base BC, find the coordinates of the points A and B. **[Foreign-2005]**

**21.** Find the coordinates of the point equidistant from the points A(1, 2), B(3, – 4) and C(5, –6).

**OR**

Prove that the points A(–4, –1), B(–2, –4), C(4, 0) and (2, 3) are the vertices of a rectangle. **[Delhi-2005C]**

**22.** Find the coordinates of the points which divide the line-segment joining the points (–4, 0) and (0, 6) in three equal parts. **[Delhi-2005C]**

**23.** Two vertices of $\Delta$ABC are given by A(2, 3) and B(–2, 1) and its centroid is G $\left(1, \frac{2}{3}\right)$. Find the coordinates of the third vertex C of the $\Delta$ABC. **[Al-2005]**

**24.** Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are the vertices of a square.

**OR**

Find the co-ordinates of the point equidistant from three given points A(5, 1), B(–3, –7) and C(7,–1) **[Delhi-2006]**

**25.** Find the value of p for which the points (–1, 3), (2, p) and (5, –1) are collinear. **[Delhi-2006]**

**26.** If the points (10, 5), (8,4) and (6,6) are the mid. Points of the sides of a triangle, find its vertices. **[Foreign-2006]**

**27.** In what ratio is the line segment joining the points (–2, –3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

**OR**

If A (5, –1), B (–3, –2) and C (–1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid. **[Delhi-2006C]**

**28.** If (–2, –1); (a, 0); (4, b) and C (1, 2) are the vertices of a parallelogram, find the values of a and b. **[Al-2006C]**

**29.** Show that the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle. **[Delhi-2007]**

**30.** In what ratio does the lines x – y – 2 = 0 divides the line segment joining (3, –1) and (8, 9) ? **[Delhi-2007]**

**31.** Three consecutive vertices of a parallelogram are (–2, 1); (1, 0) and (4, 3). Find the coordinates of the fourth vertex. **[Al-2007]**

**32.** If the point C(–1, 2) divides the line segment AB in the ratio 3 : 4 where the coordinates of A are (2, 5), find the coordinates of B. **[Al-2007]**

**33.** For what value of p, are the points (2, 1), (p, –1) and (–1, 3) collinear? **[Delhi-2008]**

**34.** Determine the ratio in which the line 3x + 4y – 9 = 0 divides joining the points (1, 3) and (2, 7). **[Delhi-2008]**

**35.** If the distances of P(x, y) from the points A(3, 6) and B(–3, 4) are equal, prove that 3x + y = 5. **[Delhi-2008]**

**36.** For what value of p, the points (–5, 1), (1, p) and (4, –2) are collinear? **[Delhi-2008]**

**37.** For what value of k, are the points (1, 1), (3, k) and (–1, 4) are collinear? **[Delhi-2008]**

**OR**

Find the area of the $\Delta$ABC with vertices A(–5, 7), B(–4, –5) and C(4, 5) **[Al-2008]**

**38.** If the point P(x, y) is equidistant from the points A(3, 6) and B(–3, 4) prove that 3x + y – 5 = 0. **[Al-2008]**

**39.** The point R divides the line segment AB, where A(–4, 0) and B(0, 6) such that AR = $\frac{2}{3}$ AB. Find the co-ordinates of R. **[Al-2008]**

**40.** The co-ordinates of A and B are (1, 2) and (2, 3) respectively. If P lies on AB find co-ordinates of P such that $\frac{AP}{PB}=\frac{3}{4}$. **[Al-2008]**

**41.** If A(4, – 8), B(3, 6) and C(5, – 4) are the vertices of a $\Delta$ABC, D is the mid point of BC and P is a point on AD joining such that $\frac{AP}{PD}=2$, find the co-ordinates of P. **[Al-2008]**

**42.** Find the value of k if the points (k, 3), (6, – 2) and (– 3, 4) are collinear. **[Foreign-2008]**

**43.** If P divides the join of A(–2, –2) and B(2, –4) such that $\frac{AP}{AB}=\frac{3}{7}$, find the co-ordinates of P. **[Foreign-2008]**

**44.** The mid points of the sides of a triangle are (3, 4), (4, 6) and (5, 7) . Find the co-ordinates of the vertices the triangle. **[Foreign-2008]**

**45.** Show that A(–3, 2), B(–5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **[Foreign-2008]**

**46.** Find the ratio in which the line 3x + y – 9 = 0 divides the line-segment joining the points (1, 3) and (2, 7). **[Foreign-2008]**

**47.** Find the distance between the points $\left(\frac{-8}{5},2\right)$ *and* $\left(\frac{2}{5},2\right)$. **[Delhi-2009]**

**48.** Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

**OR**

The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x – y + k = 0 , find the value of k. **[Delhi-2009]**

**49.** If P (x, y) is any point on the line joining the points A(a, 0) and B(0, b), then show that $\frac{x}{a}+\frac{y}{b}=1$. **[Delhi-2009]**

**50.** Find the point on x-axis which is equidistant from the points (2, – 5) and (– 2, 9) **[Delhi-2009]**

**OR**

The line segment joining the points P(3, 3), Q(6, – 6) is trisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

**51.** If the points A P(4, 3) and B(x, 5) are on the circle with the centre O (2, 3), find the value of x. **[Al-2009]**

**52.** Find the ratio in which the point (2, y) divides the line segment joining the points A(–2, 2) and B(3, 7). Also find the value of y. **[Al-2009]**

**53.** Find the area of the quadrilateral ABCD whose vertices are A(–4, –2), B(–3, –5), C(3, –2) and D(2,3). **[Al-2009]**

**54.** Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are

(0, – 1), (2, 1) and (0, 3). **[AI-2009]**

**55.** If the mid-point of the line segment joining the points P(6, b – 2) and Q(2, – 3), find the value of b. **[Foreign-2009]**

**56.** Show that the points (–2, 5), (3, –4) and (7, 10) are the vertices of a right angled isosceles triangle.

**OR**

The centre of a circle is $(2\alpha-1,7)$ and it passes through the point (–3, – 1). If the diameter of the circle is 20 units, then find the value(s) of $\alpha$. **[Foreign-2009]**

**57.** If C is a point lying on the line segment AB joining A(1, 1) and B(2, – 3) such that 3AC = CB, then find the co-ordinates of C. **[Foreign-2009]**

**58.** Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear. **[Foreign-2009]**

**59.** If the points (–2, 1), (a, b) and (4, – 1) are collinear and a – b = 1, then find the values of a and b. **[Foreign-2009]**

**60.** Find the value of K, if the points A(7, – 2), B(5, 1) and C(3, 2K) are collinear. **[AI-2010]**

**61.** Find the value of K, if the points A(8, 1), B(3, – 4) and C(2, K) are collinear. **[AI-2010]**

**62.** Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB}=\frac{K}{1}$. If P lies on the line x – y + 2 = 0, find the value of K. **[AI-2010]**

**63.** If the points (p, q), (m, n) and (p – m, q – n) are collinear, show that pn = qm. **[AI-2010]**

CO-ORDINATE GEOMETRY | **ANSWER KEY** | EXERCISE-2 (X)-CBSE

**SHORT ANSWER TYPE**

**2.** 3 : 2, – 2/5 or (4, –4) **5.** 3 : 2, 5 **6.** (–2, 0) **8.** 2/7 **9.** 3 **11.** p = 4, q = 2

**12.** (–4, –7) or A $(a/2,\ \sqrt{3a}/2)$, B(0, 0), C(a, 0) **13.** 4 : 7 or (3, 2) **14.** 2 **15.** 5 or – 3 or (3, 0)

**16.** $p=\frac{7}{3}$, q = 0 **18.** k = – 8 **19.** k = 1 **20.** $(\pm3\sqrt{3},0)$ and (0, 3) **21.** (11, 2) **22.** $\left(\frac{-8}{3},2\right),\left(\frac{-4}{3},4\right)$

**23.** (3, –2) **24.** (2, –4) **25.** p = 1 **26.** (4, 5), (8, 7), (12, 3) **27.** 2 : 3, (0, 1) or $\sqrt{65},\left(\frac{1}{3},\frac{5}{3}\right)$

**28.** a = 1 , b = 3 **30.** 2 : 3 **31.** (1 , 2) **32.** (–5, –2) **33.** p = 5 **34.** 6 : 25 **36.** – 1

**37.** – 2 or 53 sq. units **39.** $\left(-1,\frac{9}{2}\right)$ **40.** $\left(\frac{11}{7},\frac{18}{7}\right)$ **41.** (4, – 2) **42.** $k=-\frac{3}{2}$ **43.** $\left(\frac{-2}{7},\frac{-20}{7}\right)$

**44.** (4, 5), (2, 3), (6, 9) **46.** 3 : 4 **47.** 2 **48.** (0, – 2) or – 8 **50.** (–7, 0) or – 8 **51.** 2 **52.** 4 : 1, 6

**53.** 28 sq. units **54.** 1 sq. units **55.** – 8 **56.** – 4 or 2 **57.** $\left(\frac{5}{4},0\right)$ **58.** x + 3y = 7 **59.** a = 1, b = 0

**60.** 2 **61.** – 5 **62.** $\frac{2}{3}$

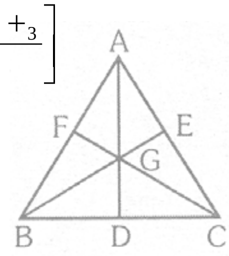
# COMPETITION WINDOW

## PROPERTIES OF TRIANGLES

Let A($x_1$, $y_1$), B($x_2$, $y_2$), C($x_3$, $y_3$) be the vertices of any $\Delta$ABC, then

**1.** **Controid :** It is the point of intersection of the cedians. It divides the median in the ratio of 2 : 1.

Co-ordinates of controid : G $\left[\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+_3}{3}\right]$




**2.** **Incentre :** It is the point of intersection of internal bisectors of the angle. Also it is the centre of the circle touching all the sides of a triangle.

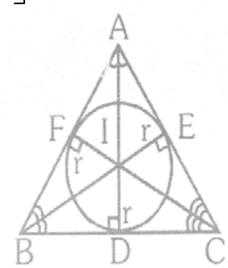
Co-ordinates of incentre $\left[\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right]$.Where a, b, c are the lengths of the sides of triangle.

The radius of incircle.

$$r = \frac{\Delta}{s}$$

Where $\Delta$ is the area of Triangle and

$$S = \frac{a+b+c}{2}$$

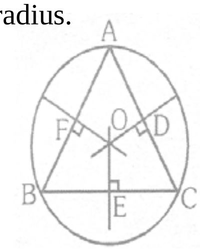



**Remark :** An angle bisector of a triangle divides the opposite side in the ratio of remaining sides.

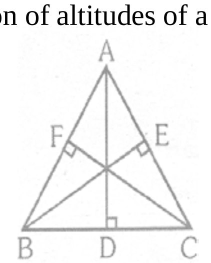
E.g. AD divides BC in the ratio $\dfrac{BD}{DC} = \dfrac{AB}{AC}$.

**3.** **Circumcentre :** It is the point of intersection of perpendicular bisectors of the sides of a triangle. It is also the centre of the circle passing through the vertices of a triangle.

If O is the circumcentre of a triangle ABC, then OA = OB = OC = circumradius.




**4.** **Orthocentre :** It is the point of intersection of altitudes of a triangle.




**Remark :**

(i) In an equilateral triangle, the centroid, incentre, orthocenter and circumcentre coincide.

(ii) In an isosceles triangle, the centroid, incentre, orthocenter and circumcentre are collinear.

(iii) In a right angled triangle, the circumcentre is the mid point of hypotenuse and the orthocenter is the point where right angle is formed.

(iv) **Euler line :** The circumcentre O, the centroid G and the orthocenter H of a triangle are collinear, the line on which they lie is called Euler line. Also G divides HO in the ratio 2 : 1.

$$\frac{OG}{GH}=\frac{1}{2}$$

### TRY OUT THE FOLLOWING

**1.** Two vertices of a triangle are (– 1, 6) are (5, 2). If its centroid is (0, – 3), find the third vertex.

**2.** If $\left(\frac{3}{2},0\right),\left(\frac{3}{2},6\right)$ and (–1, 6) are mid-points of the sides of a triangle, then find

(i) Centroid of the triangle (ii) Incentre of the triangle.

**3.** If a triangle has it's orhtocentre at (1, 1) and circumcentre at $\left(\frac{3}{2},\frac{3}{4}\right)$,then find the centriod.

### ANSWERS

**1.** (–4, –15) **2.** (i) (2/3, 4) (ii) (1, 2) **3.** (4/3, 5/6)

## COMPETITION WINDOW

### CONDITIONS FOR A TRIANGLE TO BE ACUTE, OBTUSE OR RIGHT ANGLED

For an acute angled triangle, $a^2+b^2>c^2, b^2+c^2>a^2$ *and* $a^2+b^2>c^2$.

For an obtuse angled triangle, $a^2+b^2>c^2$ (if $\angle$C is obtuse)

$a^2+c^2>b^2$ (if $\angle$B is obtuse)

$b^2+c^2>a^2$ (if $\angle$A is obtuse)

For a right angled triangle, $a^2+b^2=c^2$ (if $\angle$C = $90^0$)

$b^2+c^2=a^2$ (if $\angle$A = $90^0$)

$a^2+c^2=b^2$ (if $\angle$B = $90^0$)

Where a, b and c have their usual meanings.

## COMPETITION WINDOW

### CONCURRENCY OF LINES

Three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are said to be concurrent (lines passing through the same point) if $a_1(b_2c_3-b_3c_2)+a_2(b_3c_1-b_1c_3)+a_3(b_1c_2-b_2c_1)=0$

### TRY OUT THE FOLLOWING

**1.** Prove that the lines 3x + y – 14 = 0, x – 2y = 0 and 3x – 8y + 4 = 0 are concurrent.

**2.** Show that the lines x – y – 6 = 0, 4x – 3y – 20 = 0 and 6x + 5y + 8 = 0 are concurrent. Also find their common point of intersection.

**3.** Find the value of $\lambda$, if the lines 3x – 4y – 13 = 0, 8x – 11y – 33 = 0 and 2x – 3y + $\lambda$ = 0 are concurrent.

### ANSWER

**2.** (2, – 4) **3.** $\lambda$ = – 7

# EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

## CHOOSE THE CORRECT ONE

**1.** The circumcentre of the triangle formed by the lines xy + 2x + 2y + 4 = 0 and x + y + 2 = 0 is :
(A) (– 1, – 2) (B) (– 1, – 1) (C) (– 2, – 2) (D) (0, 0)

**2.** The vertices of a triangle (a, b – c), (b, c – a) and (c, a – b), then it's centroid lies on :
(A) y-axis (B) x-axis (C) x = 0 (D) None of these

**3.** The points (1, 2), (3, 8) and (x, 20) are collinear if x =
(A) 4 (B) 5 (C) 6 (D) 7

**4.** For the triangle whose sides are along the lines x = 0 , y = 0 and $\frac{x}{6}+\frac{y}{8}=1$, the incentre is :
(A) (3, 4) (B) (2, 2) (C) (2, 3) (D) (3, 2)

**5.** For the triangle whose sides are along the lines y = 15 , 3x – 4y = 0, 5x + 12y = 0, the incentre is :
(A) (1, 8) (B) (– 1, 8) (C) (8, 1) (D) None of these

**6.** The points D(2, 1), E(–1, –2) and F(3, 3) are the mid points of sides BC, CA and AB respectively of a $\Delta$ABC. The vertices A, B and C are :
(A) (0, 0), (6, 6), (– 2, – 4) (B) (0, 1), (6, 6), (2, 4)
(C) (1, 0), (3, 3), (– 2, – 4) (D) None of these

7. The number of integral values of m, for which x-coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is :
(A) 2 (B) 0 (C) 4 (D) 1

**8.** The radius of the circle inscribed in the triangle formed by lines x = 0, y = 0, 4x + 3y – 24 = 0 is :
(A) 12 (B) 2 (C) $2\sqrt{2}$ (D) 6

**9.** In a $\Delta$ABC, if A is the point (1, 2) and equations of the median through B and C are respectively x + y = 5 and x = 4, then B is :
(A) (1, 4) (B) (7, – 2) (C) (4, 1) (D) (– 2, 7)

**10.** The straight line 3x + y = 9 divides the segment joining the points (1, 3) and (2, 7) in the ratio :
(A) 4 : 3 (B) 3 : 4 (C) 4 : 5 (D) 5 : 6

**11.** Two opposite vertices of a rectangle are (1, 3) and (5, 1). If the equation of a diagonal this rectangle is y = 2x + c, then the value of c is :
(A) – 4 (B) 1 (C) – 9 (D) None of these

**12.** The radius of the circle passing through the point (6, 2) two of whose diameters are x + y = 6 and x + 2y = 4 is :
(A) 10 (B) $2\sqrt{5}$ (C) 6 (D) 4

**13.** The straight lines x + y = 0, 3x + y – 4 = 0, x + 3y – 4 = 0 form a triangle which is :
(A) Isosceles (B) Equilateral (C) Right angled (D) None of these

**14.** The lines segment joining the points (1, 2) and (–2, 1) is divided by the line 3x + 4y = 7 in the ratio :
(A) 3 : 4 (B) 4 : 3 (C) 9 : 4 (D) 4 : 9

**15.** If a, b, c are in A. P. then the straight line ax + by + c = 0 will always pass through a fixed point whose co-ordinates are :
(A) (1, – 2) (B) (– 1, 2) (C) (1, 2) (D) (– 1, – 2)

**16.** The lines 8x + 4y = 1, 8x + 4y = 5, 4x + 8y = 3, 4x + 8y = 7 from a :
(A) Rhombus (B) Rectangle (C) Square (D) None of these

**17.** The incentre of the triangle formed by the lines y = 15, 12y = 5x and 3x + 4y = 0 is :
(A) (8, 1) (B) (– 1, 8) (C) (1, 8) (D) None of these

**18.** The area of triangle formed by the lines y = x, y = 2x and y = 3x + 4 is :
(A) 4 (B) 7 (C) 9 (D) 8

**19.** The triangle formed by the lines x + y = 1, 2x + 3y – 6 = 0 and 4x – y + 4 = 0 lies in the :
(A) First quadrant (B) Second quadrant (C) Third quadrant (D) Fourth quadrant

**20.** A line is drawn through the points (3, 4) and (5, 6). If the line is extended to a point whose ordinate is – 1, then the abscissa of that point is ;
(A) 0 (B) – 2 (C) 1 (D) 2

**21.** The area of the triangle whose sides are along the lines x = 0, y = 0 and 4x + 5y = 20 is :
(A) 20 (B) 10 (C) $\frac{1}{10}$ (D) $\frac{1}{20}$

**22.** If a, b, c are all distend, then the equations (b – c) x + (c – a) y + a – b = 0 and $(b^3 - c^3)$ x + $(c^3 - a^3)$ y + $a^3 - b^3 = 0$ represent the same line if :
(A) a + b + c $\neq$ 0 (B) a + b c = 0
(C) a + b = 0 or b + c = 0 (D) None of these

**23.** The area of the quadrilateral with vertices at (4, 3), (2, – 1), (– 1, 2), (– 3, – 2) is :
(A) 18 (B) 36 (C) 54 (D) None of these

**24.** If $\alpha, \beta, \gamma$ are the real roots of the equation $x^3 - 3px^2 - 1 = 0$, then the centroid of the triangle with vertices $\left(\alpha \frac{1}{\alpha}\right), \left(\beta \frac{1}{\beta}\right)$ and $\left(\gamma \frac{1}{\gamma}\right)$ is at the point :
(A) (p, q) (B) (p/3, q/3) (C) (p + q, p – q) (D) (3p, 3q)

**25.** The co-ordinates of A, B C are (6, 3), (– 3, 5), (4, – 2) respectively and P is any point (x , y). the ratio of the areas of $\Delta$ABC and $\Delta$ABC is :
(A) $\left|\frac{x-y-2}{7}\right|$ (B) $\left|\frac{x+y-2}{7}\right|$ (C) $\left|\frac{x+y+2}{7}\right|$ (D) None of these

**26.** The area of a triangle is 5 square units. Two of its vertices are (2, 1) and (3, – 2). The third vertex lie on y = x + 3, the third vertex is :
(A) $\left(\frac{7}{2}, \frac{13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$ (B) $\left(\frac{7}{2}, \frac{-13}{2}\right) or \left(\frac{-3}{2}, \frac{3}{2}\right)$
(C) $\left(\frac{7}{2}, \frac{13}{2}\right) or \left(\frac{3}{2}, \frac{3}{2}\right)$ (D) None of these

**27.** The point of intersection of the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, lies on the line :
(A) x – y = 0 (B) $x + y = \frac{2ab}{a+b}$ (C) $x - y \frac{2ab}{a+b}$ (D) Both (A) and (B)

**28.** The point A divides the join of the points (–5, 1) and (3, 5) in the ratio k : 1 and co-ordinates of points B and C are (1, 5) and (7, – 2) respectively. If the area of $\Delta$ABC be 2 units, then k equals :
(A) 7, 9 (B) 6, 7 (C) $7, \frac{31}{9}$ (D) $9, \frac{31}{9}$

**29.** Q, R and S are the points on the line joining the points P(a, x) and T(b, y) such that PQ = QR = RS = ST, then $\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ is the mid point of the segment :
(A) PQ (B) QR (C) RS (D) ST

**30.** The triangle with vertices A(2, 7), B(4, y) and C(–2, 6) is right angled at A if :
(A) y = – 1 (B) y = 0 (C) y = 1 (D) None of these

**31.** The co-ordinates with of the point which divides the line segment joining (– 3, – 4) and (– 8, 7) externally in the ratio 7 : 5 are :

(A) $\left(\frac{41}{2}, \frac{69}{2}\right)$ (B) $\left(\frac{-41}{2}, \frac{-69}{2}\right)$ (C) $\left(\frac{-41}{2}, \frac{69}{2}\right)$ (D) None of these

**32.** The distance of the centroid from the orign of the friangle formed by the points (1, 1), (0, – 7) and (– 4, 0) is :

(A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$

**33.** If A(4, – 3), B(3, – 2) and C(2, 8) are vertices of a triangle, then the distance of it's centroid from the y-axis is :

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) $\frac{1}{2}$

**34.** If (5, – 4) and (– 3, 2) are two opposite vertices of a square, then it's area is :

(A) 50 (B) 75 (C) 25 (D) 100

**35.** A(6, 3), B(–3, 5), C(4, – 2) and (x, 3x) are four points. If the areas of $\Delta$DBC and $\Delta$ABC are in the ratio 1 : 2, then x is equal to :

(A) $\frac{11}{8}$ (B) 3 (C) $\frac{8}{11}$ (D) None of these

**36.** An equilateral triangle whose circumcentre is (–2, 5), one side is on y-axis, then length of side of the triangle is :

(A) 6 (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) 4

**37.** A(3, 4), and B(5, – 2) are two given points. If PA = PB and area of $\Delta$PAB = 10. then P is :

(A) (7, 1) (B) (7, 2) (C) (– 7, 2) (D) (– 7, – 1)

**38.** The distance between foot of perpendiculars drawn from a point (– 3, 4) on both axes is :

**39.** Point P divides the line segment joining A(– 5, 1) and B(3, 5) internally in the ratio $\lambda$ : 1. If Q = (1, 5), R = (7, – 2) and area of $\Delta$PQR = 2, then $\lambda$ equals :

(A) 23 (B) $\frac{29}{5}$ (C) $\frac{31}{9}$ (D) None of these

**40.** The area of an equilateral triangle whose two vertices are (1, 0) and (3, 0) and third vertex lying in the first quadrant is :

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) None of these

**41.** ABC is an isosceles triangle. If the co-ordinates of the base are B(1, 3) and C(–2, 7), the co-ordinates of vertex A is

(A) $\left(\frac{-1}{2}, 5\right)$ (B) (1, 6) (C) $\left(\frac{5}{6}, 6\right)$ (D) None of these

**42.** The area of the quadrilateral formed by the points ($a^2$ + 2ab, $b^2$), ($a^2 + b^2$, 2ab), ($a^2$, $b^2$ + 2ab) and (a2 + b2 – 2ab, 4ab) is :

(A) Zero (B) $(a + b)^2$ (C) $a^2 + b^2$ (D) $(a – b)^2$

| OBJECTIVE | | | | | | ANSWER KEY | | | | | | | | EXERCISE-4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Que.** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
| Ans. | B | B | D | C | C | A | A | B | B | B | A | B | A | D | A |
| **Que.** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| Ans. | A | C | A | B | B | B | B | A | A | B | A | D | C | B | A |
| **Que.** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** | | | |
| Ans. | C | D | C | A | A | C | B | A | C | C | C | A | | | |

# EXERCISE – 1 (FOR SCHOOL/BOARD EXAMS)

## CHOOSE THE COREECT ONE

**1.** The lines ax + 2y + 1 = 0, bx + 3y + 1 = 0 and cx + 4y + 1 = 0 are concurrent, then :
(A) a, b, c are in A.P. (B) a, b, c are in G.p. (C) a, b, c are in H.P. (D) None of these

**2.** If the lines x + 2ay + a = 0, x + 3by + b = 0 and x 4cy + c = 0 are concurrent, then a, b, c are in (abc $\neq$ 0) :
(A) A.P. (B) G.P. (C) H.P. (D) None of these

**3.** If (0, $\beta$) lies on or inside the triangle formed by the lines 3x + y + 2 = 0, 3y – 2x – 5 = 0 and 4y + x – 14 = 0 then :
(A) $\frac{5}{2} \le \beta \le \frac{7}{3}$ (B) $\frac{5}{3} \le \beta \le \frac{7}{2}$ (C) $\frac{7}{3} \le \beta \le \frac{5}{2}$ (D) None of these

**4.** If a, $x_1$, $x_2$ are in G.P. with common ratio r1 and b, $y_1$, $y_2$ are in G.P. with common ratio s where s – r = 2, then the area of the triangle with vertices (a, b), ($x_1$, $y_1$) and ($x_2$, $y_2$) is :
(A) $|ab(r^2-1)|$ (B) ab ($r^2 - s^2$) (C) ab ($s^2$ – 1) (D) abrs

**5.** If the circumcentre of a triangle lies at the origin and the centroid is the middle point of the line joining the points ($a^2$ + 1, $a^2$ + $^1$) and (2a, – 2a), then the co-ordinates of the orthocenter are :
(A) $\left[\frac{(a+1)^2}{4}, \frac{(a-1)^2}{4}\right]$ (B) $\left[\frac{3}{4}(a+1)^2, \frac{3}{4}(a-1)^2\right]$
(C) (3(a + 1)$^2$, 3 (a – 1)$^2$) (D) None of these

**6.** If every point on the line ($a_1 - a_2$) x + ($b_1 - b_2$) y = c is equidistant from the points ($a_1$, $b_1$) and ($a_2$ , $b_2$) then 2c =
(A) $a_1^2 - b_1^2 + a_2^2 + b_2^2$ (B) $a_1^2 + b_1^2 + a_2^2 + b_2^2$ (C) $a_1^2 - b_1^2 - a_2^2 - b_2^2$ (D) None of these

**7.** A rectangle has two opposite vertices at the points (1, 2) and (5, 5). If the other vertices lie on the line x = 3, the co-ordinates of the vertex nearer the axis of x are :
(A) 3, 1 (B) (3, 2) (C) (3, 4) (D) (3, 6)

**8.** If the area of the triangle formed by the pair of lines 8x$^2$ – 6y$^2$ + y$^2$ = 0 and the line 2x + 3y = a is 7, then a is equal
(A) 14 (B) $14\sqrt{2}$ (C) 28 (D) None of these

**9.** If the centroid of the triangle formed by the pair of lines 2y$^2$ + 5xy – 3x$^2$ = 0 and x + y = k is $\left(\frac{1}{18}, \frac{11}{18}\right)$, then the value of k is :
(A) – 1 (B) 0 (C) 1 (D) None of these

**10.** If $x_1$, $x_2$, $x_3$ are the abscissa of the points $A_1$, $A_2$, $A_3$ respectively where the lines y = $m_1$, x, y = $m_2$x, y = $m_3$x meet the line 2x – y + 3 = 0 such that $m_1$, $m_2$, $m_3$, are in A.P., then $x_1$, $x_2$, $x_3$ are in :
(A) A.P. (B) G.P. (C) H.P. (D) None of these

**11.** The area of the triangle with vertices $\left(1, \frac{\pi}{8}\right), \left(1, \frac{5\pi}{8}\right)$ and $\left(\sqrt{2} \frac{3\pi}{8}\right)$ is :
(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

**12.** An equilateral triangle whose orthocenter is (3, – 2), one side is on x-axis then vertex of triangle which is not on x-axis is :
(A) (3, – 6) (B) (1, – 2) (C) (9, – 2) (D) (3, – 3)

**13.** If O is the origin and the co-ordinates of A and B are $(x_1, y_1)$ and $(x_2, y_2)$ respectively then OA $\times$OB cos $\angle$AOB is equal to :
(A) $x_1y_1 + x_2y_2$ (B) $x_1x_2 + y_1y_2$ (C) $x_1y_2 + x_2y_1$ (D) $x_1x_2 - y_1y_2$

**14.** If the vertices of a triangle have integral co-ordinates, then the triangle is :
(A) Isosceles (B) Never equilateral (C) Equilateral (D) None of these

**15.** The circumcentre of the triangle formed by the points (a cos $\alpha$, a sin $\alpha$), (a cos $\beta$, sin $\beta$), (a cos $\gamma$, sin $\gamma$) is

(A) (0, 0)
(B) $\left[\left(\frac{a}{3}\right)(\cos\alpha+\cos\beta+\cos\gamma),\left(\frac{a}{3}\right)(\sin\alpha+\sin\beta+\sin\gamma)\right]$
(C) (a, 0)
(D) None of these

**16.** The x co-ordinates of the incentre of the triangle where the mid point of the sides are (0, 1), (1, 1) and (1, 0) is
(A) $2+\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $2-\sqrt{2}$ (D) $1+\sqrt{2}$

**17.** OPQR is a square and M and N are the mid points of the sides PQ and QR respectively, then ratio of area of square and the triangle OMN is :
(A) 4 : 1 (B) 2 : 1 (C) 8 : 3 (D) 4 : 3

**18.** The point with co-ordinates (2a, 3a), (3b, 2b) and (c, c) are collinear :
(A) For no value of a, b, c
(B) For all value of a, b, c
(C) If a, $\frac{c}{5}$, b are in H.P.
(D) If a, $\frac{2c}{5}$ , b are in H.P.

**19.** If co-ordinates of orthocenter and centroid of a triangle are (4, – 1) and (2, 1), then co-ordinates of a point which is equidistant from the vertices of the triangle is :
(A) (2, 2) (B) (3, 2) (C) (2, 3) (D) None of these

**20.** If the line y = mx meets the lines x + 2y – 1 = 0 and 2x – y + 3 = 0 at the number of points having integral to :
(A) – 2 (B) 2 (C) – 1 (D) 1

**21.** A triangle is formed by the point O(0, 0), A(0, 21) and B(21, 0). The number of points having integral co-ordinates (both x and y) and strictly inside the triangle is :
(A) 190 (B) 305 (C) 181 (D) 206

**22.** The straight lines 5x + 4y = 0, x + 2y – 10 = 0 and 2x + y + 5 = 0 are :
(A) Concurrent
(B) The sides of an equilateral triangle
(C) The sides of a right angled triangle
(D) None of these

**23.** A(a, b), B$(x_1, y_1)$ and C$(x_2, y_2)$ are the vertices of a triangle. If a, $x_1$, $x_2$ are in G.P. with common ratio r and b, $y_1$, $y_2$ are in G.P. with common ratio s, then area of $\Delta$ABC is :

(A) ab (r – 1) (s – 1) (s – r)
(B) $\frac{1}{2}$ ab (r + 1) (s + 1) (s – r)
(C) $\frac{1}{2}$(r – 1) (s – 1) (s – r)
(D) ab (r + 1) (s + 1) (s – r)

**24.** If a, b, c are in G.P., then the line $a^2x + b^2y + ac = 0$, will always pass through the fixed point.
(A) (0, 1) (B) (1, 0) (C) (0, – 1) (D) (1, – 1)

**25.** The lies $\ell x + my + n = 0$, $mx + ny + \ell = 0$ are concurrent if :
(A) $\ell + m n = 0$ (B) $\ell + m - n = 0$
(C) $\ell - m + n = 0$ (D) $\ell^2 + m^2 + n^2 \neq \ell m + mn + n\ell$

**26.** The sides of a triangle are 3x + 4y, 4x + 3y and 5x + 5y units where x > 0, y > 0. The triangle
(A) Right angled (B) Acute angled (C) Obtuse angled (D) Isosceles

**27.** The lines x + 2y – 3 = 0, 2x + y – 3 = 0 and the line $\ell$ are concurrent. If the line $\ell$ passes through the origin, then its equation is :
(A) x – y = 0 (B) x + y = 0 (C) x + 2y = 0 (D) 2x + y = 0

**28.** Angles of the triangle formed by the lines $x^2 - y^2 = 0$, x = 7 are :
(A) $45^0, 90^0, 45^0$ (B) $30^0, 60^0, 90^0$ (C) $60^0, 60^0, 60^0$ (D) None of these

**29.** If the orthocenter and centroid of a triangle are (– 3, 5) and (3, 3) then it's circumcentre is :
(A) (6, 2) (B) (3, – 1) (C) (– 3, 5) (D) (– 3, 1)

**30.** A triangle with vertices (4, 0), (– 1, – 1), (3, 5) is : **[AIEEE-2002]**
(A) Isosceles and right angled (B) Isosceles but not right angled
(C) right angled but not isosceles (D) Neither right angled nor isosceles

**31.** The centroid of a triangle is (2, 3) and two of it's vertices are (5, 6) and (– 3, 4). The third vertex of the triangle is : **[AIEEE-2002]**
(A) (2, 1) (B) (2, – 1) (C) (1, 2) (D) (1, – 2)

**32.** If a vertex of a triangle is (1, 1) and the mid-points of two sides through this vertex are (– 1, 2) and (3, 2), then the centroid of the triangle is : **[AIEEE-2005]**
(A) $\left(-1\frac{7}{3}\right)$ (B) $\left(\frac{-1}{3},\frac{7}{3}\right)$ (C) $\left(1,\frac{7}{3}\right)$ (D) $\left(\frac{1}{3},\frac{7}{3}\right)$

**33.** If non zero numbers a, b, c are in H.P. then the straight the $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is : **[AIEEE-2005]**
(A) (1, – 2) (B) (1, – 1/2) (C) (– 1, 2) (D) (– 1, – 2)

**34.** The line parallel to x-axis passing through the intersection of the lines ax + 2by + 3b = 0 and bx – 2ay – 3a = 0 where (a, b) $\neq$ (0, 0) is : **[AIEEE-2005]**
(A) Above x-axis at a distance 3/2 from it (B) Above x-axis at a distance 2/3 from it
(C) Below x-axis at a distance 3/2 from it (D) Below x-axis at a distance 2/3 from it

**35.** Let A(h, k), B(1, 1) and C(2, 1) be the vertex of a right angled triangle with AC it's hypotenuse. If the area of the triangle is 1, then the set of value which 'k' can take is given by : **[AIEEE-2007]**

(A) {1, 3} (B) {0, 2} (C) {– 1, 3} (D) {– 3, – 2}

**36.** The orthocenter of the right triangle with vertices $\left[2, \frac{(\sqrt{3}-1)}{2}\right], \left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(2, -\frac{1}{2}\right)$ is : **[II T-1993]**

(A) $\left[\frac{3}{2}, \frac{\sqrt{3}-3}{6}\right]$ (B) $\left[2, -\frac{1}{2}\right]$ (C) $\left[\frac{5}{4}, -\frac{\sqrt{3}-2}{4}\right]$ (D) $\left[\frac{1}{2}, \frac{1}{2}\right]$

**37.** The orthocenter of the triangle formed by the lines xy = 0 and x + y = 1 is : **[II T-1995]**

(A) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (C) (0, 0) (D) $\left(\frac{1}{4}, \frac{1}{4}\right)$

**38.** If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) not always rational points(s) ? **[II T-1998]**

(A) Centroid (B) Incentre (C) Circumcentre (D) None of these

**39.** If P(1, 2), Q(4, 6), R(5, 7) and S(a, b) are the vertex of a parallelogram PQRS, then : **[II T-1998]**

(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

**40.** The incentre of the triangle with vertex $(1, \sqrt{3})$, (0, 0) and (2, 0) is : **[II T-2000, AIEEE-2002]**

(A) $\left(1, \frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (C) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (D) $\left(1, \frac{1}{\sqrt{3}}\right)$

**41.** Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are : **[II T-2007]**

(A) $\left(\frac{4}{3}, 3\right)$ (B) $\left(3, \frac{2}{3}\right)$ (C) $\left(3, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

OBJECTIVE — ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | A | C | B | A | D | C | A | C | C | C | B | A | B | B | A |
| Que. | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Ans. | C | C | D | D | C | A | A | C | C | A | C | A | A | A | A |
| Que. | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | | | | |
| Ans. | B | C | A | C | C | B | C | B | C | D | C | | | | |